UG/I/MATH/Gen/I/15

2015

MATHEMATICS

[General]

PAPER – I (New)

Full Marks : 90

Time : 3 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

(*Classical Algebra*)

[*Marks* : 25]

1. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the product of all the values of $(1 + i)^{4/5}$. 5

(*ii*) Solve the following system of equations by Matrix method : 5

$$x + 2y - 6z = 1$$
$$2x - y + 2z = 4$$
$$x + 3y = 5$$

(*iii*) If α , β, γ are the roots of the equation $x^3 - 2x^2 + 4x - 5 = 0$, find the equation whose roots are

$$\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}.$$ 5

(*b*) (*i*) If $\tan(x + iy) = u + iv$, then prove that $u^2 + v^2 + 2u \cot 2x = 1$. 5

(*ii*) Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 4 \\ 3 & 5 & 2 \\ 4 & 8 & 0 \end{bmatrix}$. 5

(*iii*) If Δ_1 be the adjoint of a third order determinant Δ, then show that $\Delta_1 = \Delta^2$ where $\Delta \neq 0$. 5

2. Answer any *one* question : 8×1

(*a*) (*i*) If *a*, *b*, *c* are the roots of the equation $x^3 + qx + r = 0$ then find the value of $a^3 + b^3 + c^3$. 4

(*ii*) Solve the cubic equation

$$x^3 - 2x^2 - x + 2 = 0$$

by Cardan's method. 4

(*b*) (*i*) State the fundamental theorem of classical algebra. Find the value of m if the polynomial $4x^3 - 3x^2 + 2x + m$ is divisible by $(x + 2)$. Find the quotient. 2 + 3

(*ii*) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$ 3

3. Answer any *one* question : 2 × 1

(*a*) Test whether the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$ is singular or not. 2

(*b*) Solve $x^5 = 32$ by using De-Moivres theorem. 2

GROUP – B

(*Modern Algebra*)

[*Marks* : 20]

4. Answer any *two* questions : 8 × 2

(*a*) (*i*) Show that the set $G = \{1, w, w^2\}$ forms a multiplicative cyclic group whose generators are w and w^2. 4

(*ii*) Find the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 5 & 7 \\ 0 & 1 & 3 \end{pmatrix}.$$

Define eigenvalue of a matrix. 3 + 1

(*b*) (*i*) Let a, b be two elements of a group $(G, \cdot)$. Prove that $(a^{-1})^{-1} = a$ and $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$. 4

(*ii*) Show that the real quadratic form

$$x^2 + 2y^2 + 2z^2 + 2xy + 2xz$$

is positive semi-definite. 4

(*c*) (*i*) Show that the set of numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers is a field. 4

(*ii*) Give an example of a ring which is not a field. Justify your answer. 4

5. Answer any *one* question : 4 × 1

(*a*) If λ be the eigenvalue of an orthogonal matrix, then show that $\frac{1}{\lambda}$ is also an eigenvalue of that matrix. 4

(*b*) (*i*) Prove that a group contains only one identity element. 2

(*ii*) Test whether the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$$

is odd or even permutation. 2

GROUP – C

(*Analytical Geometry*)

[*Marks* : 30]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) Reduce the equation

$$x^2 + 4xy + 4y^2 + 4x + y - 15 = 0$$

into its canonical form and hence find the nature of the conic. 7

(*ii*) Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(l\cos\alpha - ep)^2 + l^2(\sin\alpha)^2 = p^2$. 8

(*b*) (*i*) Find the equation of the plane passing through the straight line

$$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$$

and perpendicular to the plane $x - y + z + 2 = 0$. 5

(*ii*) *Find the centre and radius to the* circle given by $x^2+y^2+z^2-2y-4z=11$, $x+2y+2z=15$. 5

(*iii*) Find the angle of rotation of the axes about the origin which transforms the equation $x^2-y^2=4$ to $x'y'=2$. 5

7. Answer any *one* question : 8 × 1

(*a*) (*i*) If the two curves $a_1x^2+b_1y^2=1$, $a_2x^2+b_2y^2=1$, Cut orthogonally, Prove that

$$\frac{1}{b_1}-\frac{1}{b_2}=\frac{1}{a_1}-\frac{1}{a_2}.$$ 5

(*ii*) Determine the equation of the plane containing the lines $y+2=0=z$ and $z=0=x$. 3

(*b*) (*i*) Find the length of shortest distance between the straight lines 6

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text{ and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}.$$

(*ii*) If α, β, γ be direction angles of a straight line show that

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$$ 2

8. Answer any *one* question : 4 × 1

(*a*) Find the locus of the point $P(x, y, z)$ if its distance from the straight line

$$\frac{x-1}{2} = \frac{y-2}{-4} = \frac{z-3}{2}$$ be always. 4

(*b*) Find the value of λ if the equation $\lambda xy + 16x + 28y - 14 = 0$ represents a pair of straight lines. 4

9. Answer any *one* question : 3 × 1

(*a*) Find the equation of the right circular cone whose vertex is (0,0,0) and base is the circle $y = 5, x^2 + z^2 = 16$. 3

(*b*) Show that the straight line

$$\frac{x-3}{2} = \frac{y-2}{1} = \frac{z-1}{1} = 0$$

lies in the plane $3x - 4y - 2z + 1 = 0$. 3

GROUP – D

(*Vector Algebra*)

[*Marks* : 15]

10. Answer any *one* question : 8 × 1

(*a*) (*i*) If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then prove that the vectors $\vec{a}$ and $\vec{b}$ are orthogonal. 4

(*ii*) Using vector method, prove that the diagonals of a rhombus are perpendicular to each other. 4

(*b*) (*i*) Find a unit vector which is perpendicular to both the vectors $(\hat{i}-6\hat{j}-4\hat{k})$ and $(4\hat{i}-4\hat{j}-\hat{k})$. 4

(*ii*) Prove that the lines $\vec{r} = \vec{a}+t(\vec{b}\times\vec{c})$ and $\vec{r} = \vec{b}+s(\vec{c}\times\vec{a})$ with intersect if $\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c}$. 4

11. Answer any *one* question : 4 × 1

(*a*) Using vector method prove that the angle inscribed in a semi-circle is right angle. 4

(*b*) Determine the value of λ and μ for which $-3\hat{i}+4\hat{j}+\lambda\hat{k}$ and $\mu\hat{i}+8\hat{j}+6\hat{k}$ are collinear. 4

12. Answer any *one* question : 3 × 1

(*a*) Prove that

$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$. 3

(*b*) Find the equation of a plane passing through the point (3, –2, 1) and perpendicular to the vector $4\hat{i}+\hat{j}-4\hat{k}$. 3
