OLD

2016

Part-II 3-Tier

MATHEMATICS

(General)

PAPER—III

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Linear Programming)

[Marks : 36]

1. Answer any *one* question : 1×15

(a) (i) Consider the set of equation : 5

$$4x_1 + 2x_2 - 3x_3 = 1$$
$$6x_1 + 4x_2 - 5x_3 = 1$$

Verify that $x_1 = 2$, $x_1 = 1$, $x_3 = 3$ is a feasible solution. Reduce this to a B.F.S (basic feasible solution)

(ii) An agricultural firm has 180 tons of Nitrogen fertilisers, 50 tons of Phosphate and 220 tons of Potash. It will be able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs. 15 per ton and 2:4:2 mixtures at a profit of Rs. 12 per ton. How many tons of these two mixtures should be prepared to obtain the maximum profit. 5

(iii) Solve the L.P.P. by simplex method : 5

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to
$$x_1 + 2x_2 + x_3 \le 430,$$
$$3x_1 + 2x_3 \le 460,$$
$$x_1 + 4x_2 \le 420,$$
$$x_1, x_2, x_3 \ge 0$$

(b) (i) Solve graphically the following L.P.P. 5

Minimize $z = 2x_1 + 3x_2$

Subject to
$$-x_1 + 2x_2 \le 4,$$
$$x_1 + x_2 \le 6,$$
$$x_1 + 3x_2 \ge 9,$$
$$x_1 \ge 0,\ x_2 \ge 0.$$

(ii) Prove that the set of all feasible solutions of an L.P.P. is a convex set. 5

(iii) Prove that (2,–1,0) is a solution but not a basic solution of the system of equations :

$$3x_1 - 2x_2 + x_3 = 8,$$
$$9x_1 - 6x_2 + 4x_3 = 24.$$

Find all the basic feasible solution of the above system. 5

2. Answer any *two* questions : 2×8

(a) *Using Big-M* method, solve the following L.P.P. : 8

Maximize $Z = 2x_1 - 3x_2$

Subject to
$$-x_1 + x_2 \geq -2,$$
$$5x_1 + 4x_2 \leq 46,$$
$$7x_1 + 2x_2 \geq 32,$$
$$x_1,\ x_2 \geq 0.$$

(b) Find the optimal solution of the following transportation problem : 8

	D_1	D_2	D_3	a_i
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
b_j	50	80	80	

(c) A company has five jobs to be done on five machines; any job can be done on any machine. The time in hours taken by the machines for the different jobs are as given below. Assign the machine to a job so as to minimize the total machine hours : 8

	J_1	J_2	J_3	J_4	J_5
A	11	6	14	16	17
B	7	13	22	7	10
C	10	7	2	2	2
D	4	10	8	6	11
E	14	15	16	10	18

3. Answer any *one* question : 1×3

(a) Find the dual of the following L.P.P. : 3

Minimize $z = x_1 + x_2 + x_3$

Subject to
$$\begin{aligned} x_1 - 3x_2 + 4x_3 &= 5, \\ x_1 - 2x_2 \quad\quad &\le 3, \\ 2x_2 - x_3 &\ge 4, \end{aligned}$$

$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign.

(b) Prove that the dual of the dual of a problem is the primal.

4. Answer any *one* question : 1×2

(a) Define simplex with example.

(b) State fundamental theorem of linear programming.

Group—B

(*Numerical Analysis*)

[Marks : 18]

5. Answer any *two* questions : 2×8

(a) Explain Regular Falsi method for finding a real root of the equation f (x) = 0. Give its geometrical significance. Write the advantage and disadvantage of this method. 3+3+2

(b) Find the approximate value of $\int_0^1 \frac{x}{1+x}\,dx$ (upto four decimal places) by Trapezoidal rule taking 6 equal subintervals of [0,1] and hence find the approximate value of $\log_e^2$ correct upto four decimal places. 6+2

(c) Find the value of $\sqrt{2}$ correct upto four significant figures from the following table :

x:	1.9	2.1	2.3	2.5	2.7
$\sqrt{x}$:	1.3784	1.4491	1.5166	1.5811	1.6432

6. Answer any *two* questions : 1×2

(a) If f(x) = ax, show that $\left(E + E^{-1}\right) f(x) = 2\, f(x)$

(b) Write the following numbers correct upto four significant figures :

0.00120, 520, 0.0062725, 0.090038

Group—C

(Analytical Dynamics)

[Marks : 36]

7. Answer any *one* question : 1×15

(a) (i) A particle of mass m is acted on by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 7

(ii) A particle describes an elliptic orbit under a fc which is always directed towards the centre of t ellipse. Find the law of force.

(b) (i) A particle of mass m falls from rest under gravity in a medium whose resistance is k times the velocity. Show that the distance descended in time t is $\frac{gm^2}{k^2}\left\{e^{-\frac{kt}{m}}+\frac{kt}{m}-1\right\}$. 7

(ii) A particle moves with a central acceleration $\mu\left(r+\frac{a^4}{r^2}\right)$, being projected from an apse at a distance a with a velocity $2\sqrt{\mu}a$, prove that its path is $r^2\left(2+\text{Cos}\sqrt{3}\theta\right)=3a^2$. 8

8. Answer any *two* questions : 2×8

(a) A particle describes the parabola $y^2 = 4ax$ where the force at any point is always directed perpendicularly towards its axis. Prove that force at any point must be inversely proportional to the cube of the ordinate. 8

(b) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3(u+v)x}{2(u^2+uv+v^2)}$. 8

(c) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to height h. Show that the velocity of recoil of the gun is $\left\{\frac{2m^2gh}{M(m+M)}\right\}^{1/2}$. 8

9. Answer any *one* question : 1×3

(a) A billiard ball impinges directly on an equal ball at rest. Prove that their velocities after impact are $1-e : 1+e$, where e is the coefficient of restitution.

(b) In a field of inverse square law of force, show that the velocity from infinity is $\sqrt{2}$ times the velocity in a circle at the point.

10. Answer any *one* question : 1×2

(a) A particle moves along a straight line according to the law $s^2 = 6t^2 + 4t + 3$. Prove that the acceleration varises as $\frac{1}{s^3}$. Symbols have their usual meaning.

(b) Prove that at an apse on a central orbit, the velocity is proportional to the reciprocal of the radius vector.

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