NEW

2016

Part-I 3-Tier

MATHEMATICS

(General)

PAPER—I

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(*Classical Algebra*)

[Marks : 25]

1. Answer any *one* question : 15×1

(a) (i) Show that the product of all values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}$ is 8. 5

(Turn Over)

(ii) Show that, $\text{Log}\left(\sqrt{i}\right) = \frac{1}{4}(8n+1)\pi i$. 5

(iii) Find the equation whose roots are the n-th powers of those of the equation $x^2 - 2x + 4 = 0$. 5

(b) (i) If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal roots, then show that $ab^4 - 9a^5 + c^5 = 0$. 5

(ii) Express the matrix $A = \begin{pmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{pmatrix}$ as a sum of two matrices, of which one is symmetric and the other is skew-symmetric. 5

(iii) Find A and B when $A + B = 2B^T$ and $3A + 2B = I_3$ when I_3 represents identify matrix of order 3. 5

2. Answer any *one* question : 8×1

(a) (i) Show that,

$$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a+b & a+c \\ 1 & b+a & 0 & b+c \\ 1 & c+a & c+b & 0 \end{vmatrix} = -4(ab+bc+ca).$$ 4

(ii) Solve the equation $x^4 - 9x^3 + 28x^2 + 24 = 0$ by Ferrari's method. 4

(b) (i) If $\tan \log (x + iy) = a + ib$, where $a^2 + b^2 \neq 1$, then prove that $\tan \log(x^2 + y^2) = \dfrac{2a}{1 - a^2 - b^2}$. 3

(ii) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, show that $A^2 - 2A + I_2 = 0$. Hence find A^{13327}. 5

3. Answer any *one* question : 2×1

(a) If A be a skew-symmetric matrix of order n and P be an $n\times 1$ matrix, then prove that $P^tAP = 0$. 2

(b) Use Descartes' rule of sign to show that the equation $x^8 + x^4 + 1 = 0$ has no real root. 2

Group B

(*Modern Algebra*)

[Marks : 20]

4. Answer any *two* questions : 8×2

(a) (i) Define group. Show that the set $\mathbb{Z}$ of all integers forms a group under the binary operation $*$ defined by $a * b = a + b + 1$; $a, b \in \mathbb{Z}$. Is it abelian group ? Justify. 4

(ii) If $f : A \to B$ and $g : B \to C$ be two mappings such that $gof : A \to C$ is injective, then prove that f is injective. 4

(b) (i) Define cyclic group with example. Show that every subgroup of a cyclic group is cyclic. 5

(ii) Show that every group of order 4 is abelian. 3

(c) (i) State Cayley-Hamilton theorem. Use Cayley-Hamilton theorem to compute A^{-1} where $A = \begin{bmatrix} 7 & 2 \\ 4 & 3 \end{bmatrix}$. 5

(ii) Prove that every field is an integral domain. 3

5. Answer any *one* question : 4×1

(a) (i) Show that $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 2 & 6 & 5 \end{pmatrix}$ is an even permutation and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$ is an odd permutation. 2

(ii) Let S = {1, 2, 3} and T = {1, 4, 5}. Write down a bijective mapping of S into T. 2

(b) Let G be a group and $a, x \in G$ be any two elements of G. Then, show that $O(a)$ and $O(x^{-1}ax)$ are equal. 4

Group C

(*Analytical Geometry*)

[Marks : 30]

6. Answer any *one* question : 15×1

(a) (i) If the straight lines $ax^2 + 2hxy + by^2 = 0$ are two sides of a Parallelogram and the straight line $lx + my = 1$

be one of its diagonal, then show that the equation of the other diagonal is $y(bl - hm) = x(am - hl)$. 8

(ii) Reduce the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ into its cenonical form and hence find the nature of the conic. 7

(b) (i) Find the equation of the plane which passes through the point $(2, 1, -1)$ and is orthogonal to each of the planes $x - y + z = 1$ and $3x + 4y - 2z = 0$. 5

(ii) Find the equation of the cone whose vertex is at the origin and which contains the curve given by $x^2 - y^2 + 4ax = 0, x + y + z = b$. 5

(iii) Fine the polar equation of the chord joining the two points on the parabola $\frac{2a}{r} = 1 + \cos\theta$ with $(\alpha - \beta)$ and $(\alpha + \beta)$ as their vectorial angles. 5

7. Answer any *one* question : 8×1

(a) (i) If the normal be drawn at one extremity $\left(l, \frac{1}{2}\pi\right)$ of the latus rectum PSP$'$ of the conic $\frac{l}{r} = 1 + e\cos\theta$ where S is the Pole, then show that the distance from the focus S of the other point in which the normal meets the conic is $\frac{l\left(1 + 3e^2 + e^4\right)}{1 + e^2 - e^4}$. 6

(ii) Show that the three points $(-1, 5, 3)$, $(5, 1, 5)$ and $(8, -1, 6)$ are collinear. 2

(b) (i) Show that straight lines whose directions cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles. 4

(ii) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point $(2, -4, 6)$. 4

8. Answer any *one* question : 4×1

(a) (i) If r_1 and r_2 are two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2 \cos^2 \theta}$, prove that

$$\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$ 2

(ii) If PSP' be the focal chord of a conic. Show that $\frac{1}{SP} + \frac{1}{SP'} = \frac{2}{l}$, where l is the semi-latus rectum. 2

(b) Find the equation of the cylinder whose generaling line is parallel to the z-axis and the guiding curve is $x^2 + y^2 = z, x + y + z = 1$. 4

9. Answer any *one* question : 3×1

(a) Find the bisectors of the angles between the straight lines $2x^2 + 5xy + 2y^2 + 15x + 18y + 28 = 0$. 3

(b) Find the locus of the poles of chords of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ *whose mid-point* lies on the straight line x + y = 1. 3

Group D

(*Vector Algebra*)

[Marks : 15]

10. Answer any *one* question : 8×1

(a) (i) For any three vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ prove that

$$\vec{\alpha}\times\left(\vec{\beta}\times\vec{\gamma}\right)=\left(\vec{\alpha}.\vec{\gamma}\right)\vec{\beta}-\left(\vec{\alpha}.\vec{\beta}\right)\vec{\gamma}.$$ 4

(ii) If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ is such that $\vec{a}+\lambda\vec{b}$ is perpendicular to c, then find the value of λ. 4

(b) (i) If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and acute angle θ with $\hat{k}$, then find the value of θ and hence find the components of $\vec{a}$. 4

(ii) Find the area of the triangle with ventices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5) using vector method. 4

11. Answer any *one* question : 4×1

(a) Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \left[\vec{a},\ \vec{b},\ \vec{c}\right]^2$. 4

(b) Find $\left|\vec{x}\right|$, if $\vec{p}$ is a unit vector and $\left(\vec{x}-\vec{p}\right)\cdot\left(\vec{x}+\vec{p}\right) = 80$.

4

12. Answer any *one* question : 3×1

(a) If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ then show that $\left(\vec{a}-\vec{d}\right)$ is parallel to $\left(\vec{b}-\vec{c}\right)$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

3

(b) Find the equation of the plane through the points (3, 2, 0), (1, 3, −1) and (0, −2, 3). 3

———