2017

M.Sc. 4th Seme. Examination

PHYSICS

PAPER—PHS-401

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

Group-A

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Show that acceleration is a contravariant vector.

(b) Prove that the ordinary derivative of a vector is not a tensor but its covariant derivative is a tensor.

(c) Show that the divergence of $\left(R^{\mu}_{\nu} - \frac{1}{2}g^{\mu}_{\nu}R\right)$ is identically zero.

(d) What is value of $[\lambda\mu, \nu] + [\lambda\nu, \mu]$?

(e) Show that $A^{\nu}_{,\nu} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x_{\nu}}\left\{\sqrt{g}A^{\nu}\right\}$, where g is $\det(g_{\mu\nu})$.

(f) The mass density of photons and neutrions at 3000K is 9.9×10^{-22} gm/c.c and pressure is 0.3 gm/cm sec^2. Calculate Jeans mass.

(g) A star has luminosity is $100L_{\theta}$ and apparent bolometric magnitude $m^{bol}_{star} = 9.7$. If the sum has $m^{bol}_{\theta} = +4.7$, calculate the distance of the star.

(h) Explain the formation of neutron star and why relativistic corrections are not significant for white dwarfs.

2. (a) Obtain the non-vanishing Christoffel's symbols for the metric

$$ds^2 = f(x)dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}dt^2$$

and hence derive the differential equations of geodesies for this surface. 6

(b) The R-W metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2.d\Omega^2\right)$$

where $k = 0, +1, -1$, according to whether the 3-D space has zero, positive or negative curvature respectively.

Einstein field equation

$\dot{a}^2 + k = \frac{8\pi GP}{3}a^2$ for a matter-dominated universe of density ρ.

(i) Calculate the distance $L_r(t)$ from the origin (r = 0) to a particle with co-ordinate r at time t in terms of r and a(t), also show that $\ddot{a} = -\frac{4\pi G\rho}{3}a$. 2+2

3. (a) If universe is described by R-W line element and a spaceship sets out with velocity v relative to cosmological observers. At a latter time when the universe has expanded by a scale factor (1 + z). Show that velocity with respect to cosmological observers is $\frac{v}{(1+z)}$.

(b) Show that $R_{11} = \frac{-\left(a\ddot{a} + 2\dot{a}^2\right)}{1 - kr^2}$ for the line element described in 2(b). 5+5

Group-B

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Consider a degenerate (i.e. T = 0K) gas of N non-interacting electrons in a volume V. Find an equation relating pressure, energy and volume of this gas for the extreme relativistic case. (ignore the electron mass)

(b) If particle masses and chemical potential are neglected compared with $k_B T$ then write down the average number and energy densities of a gas of non-interacting fermions in thermal equilibrium under these conditions.

(c) State Onsager reciprocity theorem.

(d) Compare B-E condensation and λ-transition.

(e) Draw the chemical potential vs. temperature in case of B-E and F-D gases.

(f) $G(m, T) = G_0(T) + \frac{a}{2}m^2 + \frac{b}{4}m^4 + \cdots$.

Find the possible solutions of m in context of Landau theory of second order phase transition.

(g) Write down the relation between specific heat and temperature for photon, phonon and material particle. Can you identify the general rule in these expressions?

(h) Define spin-spin correlation function. How susceptibility is related with it?

2. Consider a crystalline lattice with Ising spin $S_{\vec{l}} = \vec{l}$ at site $\vec{l}$. In the presence of an external field $\vec{H} = (0, 0, H_0)$, the Hamiltonian of the system may be written as

$$H = -J_e \sum_{l, l'} S_l S_{l'} - \mu_0 H_0 \sum_{l} S_l, \text{ where } J_e > 0 \text{ is a constant}$$

and the $\sum_{l, l'}$ is over all nearest-neighbour sites only (each site has z nearest neighbours).

(i) Write an expression for the free energy of the system at temperature T.

(ii) Using the MFA, derive an equation for the spontaneous magnetization $m = \langle S_0 \rangle$ for $\vec{H}_0 = 0$ and calculate the critical temperature T_c below which $m \neq 0$.

(iii) Calculate the critical exponent β defined by

$$m\left(T,\ \vec{H}_0 = 0\right) \sim \text{const.}\left(1 - \frac{T}{T_c}\right)^{\beta} \text{ as } T \to T_c.$$

(iv) Describe the behaviour of the specific heat at constant $\vec{H}_0$, $C\left(\vec{H}_0 = 0\right)$ near $T = T_c$. 2+3+2+3

3. (a) Derive an expression of photo-electric current density for a metal.

(b) Show that average energy per particle for an ideal gas of N spin $\frac{1}{2}$ fermions confined in a volume V in an external magnetic field H is $\frac{E}{N} \simeq \frac{3}{5}\mu(0)\left[1-\frac{5}{2}\left(\frac{\mu_B H}{\mu_0}\right)^2\right]$.

(c) The pressure $P = \frac{2}{5}n\mu(0)$

and susceptibility $X = \frac{3}{2}\ \frac{N\mu_B^2}{\mu(0)V}$. 4+3+3

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