2017

M.Sc.

1st Semester Examination

PHYSICS

PAPER—PHS-101

Subject Code—33

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A and Group-B

(Methods of Mathematical Physics)

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Find the principal value of the integral $\int_{-\infty}^{+\infty} \frac{\sin(2x)}{x^3} dx$.

(Turn Over)

(b) Show that for large n and small θ, $p_n(\cos\theta) = J_0(n\theta)$.

(c) Show that for Laguerre's polynomial $L_n(o)$ = n!.

(d) Find the Laurent series about the singularity for the function $\frac{e^z}{(z-2)^2}$.

(e) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$; 0<n<1

(f) Prove that $|\Gamma(in)|^2 = \frac{\pi}{n \sinh(n\pi)}$.

(g) Show that determinant of a matrix remains invariant under orthogonal similarity transformation.

(h) Consider P be a $n \times n$ diagonalizable matrix which satisfies the equations $P^2 = P$, $T_r(P) = n-1$. Find det.(P).

2. (a) Consider the vectors (2, −1, 2), (1, 1, 4) and (6, 3, 9). Use the Gram-Schmidt orthogonalization procedure to find orthogonal vectors.

(b) Consider two different sets of orthogonal basis vectors $S = \left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $S' = \left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two dimensional real vector space. Find a matrix P, for changing basis from S to S'.

(c) Evaluate $\int_0^\infty \frac{dx}{x^6+1}$ by Cauchy's residue theorem.

3+3+4

3. (a) Evaluate $\frac{1}{2\pi i}\oint_c \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$ around the unit circle traversed in the anti-clockwise direction.

(b) If m be a positive integer, then show that for $A=\begin{pmatrix}3 & -4\\ 1 & -1\end{pmatrix}$.

The value of $A^m=\begin{pmatrix}1+2m & -4m\\ m & 1-2m\end{pmatrix}$.

(c) If (θ_1,ϕ_1) and (θ_2,ϕ_2) are two different direction in spherical polar coordinates and θ is the angle between these two directions, then prove that

$$P_l(\cos\theta)=\frac{4\pi}{2l+1}\sum_{m=-l}^{+l} Y_{lm}(\theta_1,\phi_1)Y_{lm}^{*}(\theta_2,\phi_2)$$

4+3+3

Group—B

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *four* of the following : $4 \times 2\frac{1}{2}$

(a) Prove that Possion's Bracket remain invarint under canonical transformation.

(b) A particular mechanical system depending on two coordinates u and v has kinetic energy $T = v^2\dot{u}^2 + 2\dot{v}^2$ and potential energy $V = u^2 - v^2$. Write down the Lagrangian for the system and deduce its equation of motion. (do not attempt to solve them).

(c) A particle moves in a plane under the influence of a force, whose magnitude is $F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2\ddot{r}r}{c^2}\right)$ where r is the distance of the particle to the centre of force. Find the potential that will result in such a force, and from that the Lagrangian for the motion in a plane.

(d) What kind of transformaion is generated by the function

$F = -\sum_i Q_i p_i$?

(e) Explain Exchange transformation and Identity transformation.

(f) Show that $\Delta \int_{t_1}^{t_2} \Sigma p_k \dot{q}_k dt = \delta \int_{t_1}^{t_2} L dt + (L + H)[\Delta t]_{t_1}^{t_2}$

2. (a) Explain Hamilton's principle.

(b) For a dynamical system having q_k and p_k respectively the generalised co-ordinates and momenta and Hamiltonian H, derive the following relations ;

$$\dot{q}_k = \frac{\partial H}{\partial p_k} \quad \text{and} \quad \dot{p}_k = -\frac{\partial H}{\partial q_k}.$$

(c) For a system consisting of a single particle show that the principle of least action becomes,

$$\Delta \int \sqrt{H - V}, ds = 0$$

where ds = elementary path, H = Hamiltonian and V = Potential energy. 2+4+4

3. (a) Determine the oscillations of a system with two degrees of freedom whose Lagrangian is,

$$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}w_0^2(x^2 + y^2) + \alpha xy.$$

(b) For the Hamiltonian $H = \frac{(p^2 + q^2)}{2}$. Find $[\dot{p}, H]$ and $[\dot{q}, H]$ and find the values of p and q. Also show that the energy is constant. 5+5
