2017

M.Sc. 2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-203

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Notations and symbols have their usual meaning.

(Abstract Algebra and Linear Algebra)

Unit-I

(Abstract Algebra)

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Show that any ring of order 6 is commutative.

(b) Define a unique factorization domain with an example.

(Turn Over)

(c) Let G be any group and A = G. Show that the map defined by $g.a = ag^{-1}$, $\forall$ a, g $\in$ G satisties the axiions of a group action of G on itself.

2. (a) State and prove Cayley's theorem for a group.

(b) Let G = (R, +), H = (Z, +) and G′ = ({z $\in$ C / $|z| = 1$}, ·) be the groups. Prove that $\frac{G}{H} \cong G'$. 4+4

3. (a) Write down the class equation for finite groups. 2

(b) State Sylow's third theorem. Show that no group of order 63 is simple. 2+2

(c) Let G be a finite non-commutative group. Show that $|Z(G)| \leq \frac{1}{4}|G|$, where Z(G) is the centre of the group G. 2

4. (a) Define Enclidean domain with an example. Show that the domain D = $Z[\sqrt{-5}]$, but a unique factorization domain.

(b) Let R be a commetative ring with unity. Show that an ideal P is a prime ideal iff the quotient ring $R/_P$ is an intigral domain. 5+3

[*Internal Assessment* : 5]

Unit-II

(Linear Algebra)

[Marks : 25]

Answer Q. No. 5 and any *two* from the rest.

5. Answer any *two* questions : 2×2

(a) Fill in the blanks :

(i) A linear transformation T on $\mathbb{R}^2$ defined by T(x, y) = (ax + by, cx + dy) will be investible iff ....

(ii) A linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that T(x, y) = (x – y, y) then $T^2(x, y)$ = ... 1+1

(b) Let V be a finite dimensional vector space. What is the minimal polynomial for the identity operator on V ? What is the minimal polymonial for the zero operator ? 1+1

(c) Define Jordan block with an example. 2

6. (a) Find the minimal polynomial for the real matrix

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$ 5

(b) Let T be a linear map on $V_3(\mathbb{R})$ defined by T(a, b, c) = (3a, a – b, 2a + b + c) $\forall$ a, b, c $\in \mathbb{R}$.
Is T invertible ? If so, find a rule for T^{-1} like one which defines T. 3

7. (a) Show that the map $T : V_2(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ defined by $T(a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(\mathbb{R})$ into $V_3(\mathbb{R})$. Find the range, rank, null space and nullity of T. 4

(b) Give the definition of a lattice with respect to poset and also give the definition of a lattice with respect to an algebra. Show that the two definitions an equivalent. 4

8. (a) Prove that a linear operator has a diagonal matrix representation if its minimal polynomial is a product of distinct linear polynomials. 4

(b) Let T be a linear operator on $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. What is the matrix representation of T relative to the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (-1, 2, 1)$, $\alpha_3 = (2, 1, 1)$? 4

[Internal Assessment —5]

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