2017

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-105

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Classical Mechanics and Non-linear Dynamics]

Answer Q. No. *1* and
any *four* questions from the rest.

1. Answer any *four* questions : 4×2

(a) Show that for conservative holonomic system,

$$\frac{\partial L}{\partial \dot{q}_j} = \int \frac{\partial L}{\partial q_j} dt.$$

(b) Show that the expression $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz's transformation.

(c) What do you mean by bifurcation of a system ?

(d) Find the condition such that the transformation $P = ap + bq$, $Q = cp + dq$ is canonical.

(e) If the generating function is known then show that the canonical transformation can be determined.

(f) Define action and state Hamilton's principle.

2. (a) Show that Lagrange's equations of motion remain unchanged in form if you add to the Lagrangian a total time derivative of any function of generalised coordinates and time. 4

(b) The potential energy and kinetic energy of a dynamical system are given by

$$V = \frac{1}{2}kr^2$$

and

$$T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}mr^2\sin^2\phi\dot{\phi}^2.$$

Determine the Lagrangian and Lagrange's equations of motion. 4

3. (a) Show that the Poisson bracket is invariant under canonical transformation. 3

(b) Show that $E = mc^2$, in relativistic mechanics. 5

4. State Hamilton's principle and derive it from D'Alembert's principle. 8

5. Show that the Coriolis force due to the rotation of earth deflects a vertically falling particle in northern hemisphere toward east and the deflection is proportional to $h^{3/2}$ for a given latitude where h is the height of the fall. 8

6. Solve the following dynamical system

$$\dot{x}_1 = -x_1 - 3x_2, \quad \dot{x}_2 = 2x_2.$$

Also, sketch the phase portrait. 8

7. Consider the equilibrium configuration of the molecule such that two of its atoms of each of mass M are symmetrically placed on each side of the third atom of mass m. All three

atoms are collinear. Assume the motion along the line of molecules and there being no interaction between the ends atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms. 8

(Internal Assessment : 10 Marks)
