2017

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-102

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Complex Analysis)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) If $f(z)$ is analytic, then show that $f'(z) = (\cos\theta - i \sin\theta)\frac{\partial f}{\partial r}$, where $z = re^{i\theta}$.

(b) Find the branch cut of $\text{Log}(z + 2i)$.

(c) Is it possible to evaluate the integral $\int_C f(z)dz$, where $f(z) = (5z + 2) / \{z(z - 2)\}$ and $C : |z| = 1$, using the single residue of $\frac{1}{z^2} f\left(\frac{1}{z}\right)$ at $z = 0$? Justify.

(d) State the Laurents' theorem.

(e) A linear transformation with two distinct fixed points α and β can be put in a form $\frac{w - \alpha}{w - \beta} = k \frac{z - \alpha}{z - \beta}$ where k is constant. Under what value/s of k, the above transformation is elliptic, hyperbolic and loxodromic ?

(f) Let C be any simple closed contour, described in the positive sense at the z-plane and let $g(w) = \int_C \frac{z^3 + 2z}{(z - w)^3} dz$. Then find $g(w)$, when w is inside C.

2. (a) Without evaluating the integration, find an upper bound of the integral

$$\int_C \left(\frac{e^{2z} - \frac{\sqrt{3}}{2} \bar{z}}{z^2 + 2} \right) dz$$, where C is the arc of the circle $|z| = \sqrt{3}$ from $z = -\sqrt{3}$ to $z = -i\sqrt{3}$, taking in anti-clockwise direction.

(b) Construct a complex function which is continuous everywhere but nowhere analytic. Justify your answer. 6+2

3. (a) Using an antiderivative, evaluate the integral

$$\int_{1-i\sqrt{3}}^{1+i\sqrt{3}} \left(\frac{5\pi}{z} + 3iz^{i-1} \right) dz$$

by taking any path of integration in region $y < \sqrt{3}x$ taken from $z = -1 - \sqrt{3}i$ to $z = 1 + \sqrt{3}i$, except for its end points. (Use principal branches of the required functions.)

(b) Find the order of the pole of the function $f(z) = \dfrac{1}{\cos z - \sin z}$ at $z = \dfrac{\pi}{4}$. 5+3

4. (a) Let $f(z) = (x^3 + 2) + i(1 - y)^2$. Find all the points in the complex plane where $f(z)$ is differentiable and then compute $f'(z)$ at those points. Is $f(z)$ analytic at any point in the complex plane ? Justify.

(b) Find Taylor or Laurent series expansion of the function $f(z) = \dfrac{3}{z(z-i)}$ with centre at $c = -i$, where the region of convergence is $1 < |z + i| < 2$. 4+4

5. (a) Classify the singularity at $z = 0$ if the function $f(z) = \frac{\cosh(z^3) - 1}{z^7}$ in terms of removal singularity, pole and essential singularity.

   (b) Evaluate $\int_C \frac{\cosh(z^3) - 1}{z^7} dz$, where $C : |z| = 1$ taken in the positive direction. 4+4

6. (a) State and prove the Cauchy's theorem.

   (b) Find a conformal map of the unit disk $|z| < 1$ onto the right half-plane $Re(w) > 0$. 4+4

7. (a) Using the method of residues, evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

   (b) State the Jordan's Lemma. 5+3

(Internal Assessment : 10 Marks)

———