2016

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-105

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Classical Mechanics and Non-linear Dynamics]

Answer Q. No. *1* and any *four* from questions from the rest.

1. Answer any *four* questions : 4×2

(a) Show that the rate of change of angular momentum is equal to the applied torque for a system of particles.

(b) Show that the distribution law over addition is valid for Poisson bracket.

(c) Define Lagrangian and Hamiltonian of a dynamical system.

(d) Find the Kinetic energy of a body when it is rotating about a fixed point.

(e) State Hamilton principle. What is the importance of this principle ?

(f) Define Hamilton-Jacobi's equation.

2. (a) If a transformation from p, q to P, Q be Canonical them show that $\sum_i (\delta p_i dq_i - \delta q_i dp_i)$ is invariant. 4

(b) Suppose, $q = \sqrt{\frac{P}{\sqrt{k}}} \sin Q,\ p = \sqrt{mP\sqrt{k}} \cos Q$.

Is the transformation Canonical ? If yes, find the generating function G (p, Q). 4

3. Derive Lorentz transformation equations. 8

4. Three oscillators of equal masses are coupled in such a manner that the potential energy of the system is

$$V = \frac{1}{2}\left[K_1\left(q_1^2 + q_2^2 + q_3^2\right) + K_2 q_2^2 + K_3\left(q_1 q_2 + q_2 q_3\right)\right].$$

Find the eigen frequencies. 8

5. A top has an axis of symmetry OG, where G is the centre of mass and it spins with the end O on a rough horizontal table. The mass of the top is m and its moment of inertia about OG and any axis through O perpendicular to OG are C and A respectively. Initially, OG is vertical and the top is set spinning with spin n about its axis. It is then slightly displaced. If in the subsequent motion, θ is the angle OG makes with the vertical and ϕ is the angular velocity about the vertical, show that

$A\dot{\phi}\sin^2\theta = cn(1 - \cos\theta)$ and $A(\dot{\phi}^2\sin^2\theta + \theta^2) = 2mgh(1 - \cos\theta)$

where OG = h. 8

6. (a) Deduce the Euler-Poisson equation to find the curve $y = f(x)$ such that the functional

$$J = \int_{x_0}^{x_1} F\left(y(x), y'(x), y''(x), \ldots, y^{(n)}(x), x\right)dx$$

is optimum. 4

(b) Derive the differential equations of the lines of propagation of light in an optically non-homogeneous medium with the speed of light $C(x, y, z)$. Also, discuss the case when C is constant. 4

7. A particle of mass m_2 is suspended by a light inextensible string of length l and another particle of mass m_1 at the point of support of m_2 and it can be moved on a horizontal line lying in the plane in which m_2 moves.

Calculate the Lagransian function and solve the above problem. 8

(Internal Assessment : 10 Marks)

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