2016

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-304

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Special Paper : Dynamical Oceanology–I / Advanced Optimization and Operations Research]

(Dynamical Oceanology–I)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) Show the contour plot of density as a functions of temperature and salinity over ranges appropriate to most of the ocean.

(b) Write the expression for horizontal coriolis acceleration and find its value at latitude 90°, 45° & 0°.

(Turn Over)

(c) Write the properties of Reynolds operator of averaging.

(d) Define all the Ekman numbers as well as Rossby number.

(e) From the Ekman observation, write the relation between the Ekman depth and wind speed and then calculate the depth at latitude 45° for wind speed 12 m/s.

(f) Write the x- and y- components of Reynolds Averaged Navier Stokes (RANS) equation.

2. (a) Write an equations of motion (components form) in oceanography.

(b) Derive the pressure term in vector form for the above equation.

(c) Write the transformation in vector form ideal axes fixed in space to axes fixed to earth, then using this transformation, derive the equation of motion in axes fixed to earth. 2+2+4

3. Derive the Reynolds equation for the x-component of velocity. 8

4. (a) For the ocean with horizontal and vertical length scales 10^3 KM and 1 KM, respectively and horizontal speed of order 0.1 m/s, find the vertical velocity and values of all eddy viscosities.

(b) For the above situation, scale the y- and z- momentum equations and hence derive the corresponding approximate equations. 4+4

5. (a) Derive the expression for geopotential distance between two levels Z_1 and Z_2.

(b) With the necessary assumptions, derive the geostrophic equations and hence combine these to a single equation.

4+4

6. (a) With the necessary assumptions, derive the equation for 'β'-spiral.

(b) Hence derive the following relation

$$u\frac{\partial^2 h}{\partial x \partial z} + v\left(\frac{\partial^2 h}{\partial y \partial z} - \beta/f\right) = 0$$

with usual notation. 5+3

7. (a) Find the Ekman's solution of the equation of wind-driven circulation with friction present.

(b) Derive the suerdrup equation for motion of wind-driven circulation with friction present.

(Internal Assessment : 10 Marks)

(Advanced Optimization and Operations Research)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions of the following : 4×2

(a) Discuss when dual simplex method is useful over simplex method. 2

(b) Discuss the effect of addition a new variable in the optimal table of simplex problem. 2

(c) State the necessary and sufficient conditions for an extreme point of a n-variable function. 2

(d) What do you mean by sensitivity analysis ? 2

(e) Explain uni-modal maximization function with an example. 2

(f) Discuss the different types of achievement of objective in goal programming. 2

2. Discuss the steps of revised simplex method to solve a LPP. Write down the salient points of the difference of revised simplex and simplex method. 5+3

3. Solve the following Integer Programming Problem usin; Gomary's cutting plane method

$$\text{Maximize } z = 5x_1 + x_2$$
$$\text{subject to } 8x_1 + 6x_2 \le 15$$
$$2x_1 \le 3$$
$$x_1, x_2 \ge 0$$
$$x_1, x_2 \text{ are integers.}$$

8

4. Solve the following LPP by artificial constraint method

$$\text{Min } Z = -2x_1 - x_2 - x_3$$

$$\text{Subject to } 4x_1 + 6x_2 + 3x_2 \le 8$$

$$-x_1 + 9x_2 - x_3 \ge 3$$

$$2x_1 + 3x_2 - 5x_3 \ge 4$$

$$x_1, x_2, x_3 \ge 0$$ 8

5. Using steepest descent method minimize

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 2gx_1 + 2fx_2 + 2hx_3 + c$$

starting from the point $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$. 8

6. Maximize $f(x) = \begin{cases} 3(x-1), & 1 \le x \le 2 \\ 3(4-x)/2, & 2 \le x \le 4 \end{cases}$

in the interval [1, 4] using Fibonacci method for five experiments. 8

7. Solve the following goal programming problem

$$\text{Min } Z = P_1 d_6^+ + P_2\left(2d_2^- + d_3^-\right) + P_3 d_1^-$$

Subject to the constraints

$$20x_1 + 10x_2 + d_4^- - d_4^+ = 60$$

$$10x_1 + 10x_2 + d_5^- - d_5^+ = 40$$

$$40x_1 + 80x_2 + d_1^- - d_1^+ = 1000$$

$$x_1 \qquad + d_2^- - d_2^+ = 4$$

$$x_2 + d_3^- - d_3^+ = 6$$

$$d_4^+ + d_5^+ + d_6^- - d_6^+ = 50$$

$$x_1, x_2, d_i^-, d_i^+ \geq 0 \; ; \; i = 1, 2, 3, 4, 5, 6.$$ 8

(Internal Assessment : 10 Marks)
