2015

M.Sc. Part-II Examination

PHYSICS

PAPER—VIII

Full Marks : 75

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate answer-scripts for Group-A and Group-B

Group-A

(Advanced Quantum Mechanics)

[ Marks : 40 ]

Answer Q. No. 1, 2, 3 and two from the rest.

1. Answer any five bits :

   (i) If the doublet splitting of the first excited state

      \[ {}^2P_{3/2} \rightarrow {}^2P_{1/2} \]

      of He\(^+\) is 5.84 cm\(^{-1}\), calculate the

      \[ \hbar \]
corresponding separation of Hydrogen atom.

(ii) Prove the optical theorem

$$\sigma_{\text{total}} = \frac{4\pi}{k} \text{Im} \left[ f(0) \right]$$

where the symbols carry their usual meaning.

(iii) Indistinguishability of identical particles lead to symmetric or antisymmetric nature of wave-functions — Justify.

(iv) Consider a system of two identical particles each with spin $\frac{1}{2}$. The Hamiltonian of the system is given as

$$\hat{H} = A + B\hat{S}_1 \cdot \hat{S}_2 + C(\hat{S}_{1z} + \hat{S}_{2z})$$

Find the eigenvalues of the system.

(v) Express Dirac equation in covariant form and express the properties of $\gamma$ matrices.

(vi) Show that

$$\left( \bar{a} \cdot \hat{A} \right) \left( \bar{a} \cdot \hat{B} \right) = \hat{A} \cdot \hat{B} + i\sigma^d \left( \hat{A} \times \hat{B} \right)$$

(vii) Obtain eigenvalues of the operator

$$K = \frac{\hat{r}}{k} \left( \hat{S} \cdot \hat{L} + \hat{h} \right)$$

(viii) Derive the continuity equation for spin $\frac{1}{2}$ particles and explain the terms.

2. Answer any two bits:

(i) Find the scattering cross-section for scattering of a particle of mass $m$ by the $\delta$-function potential

$$V(r) = g \delta(r)$$

where $g = \text{constant}.$

(ii) Prove that

$$\left( \hat{a} \cdot \hat{B} \right) \left( \hat{a} \cdot \hat{C} \right) = 4\hat{B} \cdot \hat{C}.$$  

(iii) Show that

$$\left[ \hat{S}, S_r \right] = i\hbar \frac{\hat{r} \times \hat{S}}{r}$$

where $S_r = \frac{\hat{s} \cdot \hat{r}}{r}$.

3. Answer any one bit:

(i) Three non-interacting identical Fermions are in an infinite potential well denoted by

$$V(x) = 0 \quad \text{for} \quad 0 < x < a$$

$$= \infty \quad \text{for} \quad x < 0 \text{ and } x > a$$

What would be the ground state energy?
(ii) Find the C.G. coefficients for $j_1 = j_2 = \frac{1}{2}$.

4. (a) Establish the expression of a plane wave in terms of spherical waves.

(b) In the partial wave analysis of scattering find the criterion for determining the significant number of spherical waves.

(c) Obtain an expression for the phase shift $\delta_0$ for S-wave scattering by the potential

$$V(r) = \begin{cases} \infty & \text{for } 0 \leq r \leq a \\ 0 & \text{for } r > a \end{cases}$$

Show that form factor $F(q)$ for high energy elastic scattering is given by

$$F(q) = \left(\frac{3}{q^2 R^2}\right) \left(\frac{\sin qR}{qR} - \cos qR\right)$$

where $q = \text{momentum transfer wave vector}$. 

(b) Describe Fermi-Thomas model of the atom and prove that

$$\frac{d^2 \chi}{dx^2} = \chi^{3/2} \frac{x^{1/2}}{\chi^{1/2}}$$

5. Obtain Dirac equation for a free particle and obtain its solution. Discuss various implication of negative energy states. 

5+2+3

5. (a) If $V(r) = -\frac{ze^2}{2R} \left(3 - \frac{r^2}{R^2}\right)$ for $0 < r < R$

$$= -\frac{ze^2}{r} e^{-ar} \quad \text{for } R < r < \infty$$

Group-B

(Statistical Mechanics)

[ Marks : 35 ]

Answer Q. No. 1 and two from the rest.

1. Answer any five bits : 

(a) A system of three cells such that $N_1 = 5$, $N_2 = 3$, $N_3 = 2$; $E_1 = 0$, $E_2 = 2$, $E_3 = 4$ joules per particle. If total no. of particles and energy are constant and $\delta N_3 = -2$ then find $\delta N_1$ and $\delta N_2$. 

C/16/DDE/M.Sc./Part-II/Physics/8 (Continued)
2. (a) Prove that two-dimensional ideal B-E gas cannot undergo B-E condensation.

(b) Write down the expression for free energy of FD gas under magnetic quantization. Prove that degree of degeneracy is given by

$$g = \frac{L_x L_y H}{\left(\frac{hc}{e}\right)}$$

for a two-dimensional system of dimension $L_x, L_y$ with magnetic field $H$.

(c) Write down an expression for isothermal susceptibility according to G-L theory of phase transition and explain all the terms.

3. (a) Define long range and short range order parameter.

(b) Prove that temperature dependence of long range order parameter for Ising-spin system in a magnetic field $\tilde{H} = \hat{e}_z H$ is given by

$$L(T) = \tanh \beta (J_y L + \mu_0 H)$$

where $\gamma = \text{no. of n.n. and other symbols have usual meanings.}$
Also prove that
\[ L = \frac{\sqrt{3(T_C - T)}}{T_C} \]
near transition temperature. 

4. (a) Deduce an expression of B-E distribution function from
grand partition function.

(b) Prove that the average dipole-magnetic moment
\[ \langle \mu_z \rangle = g m_j \mu_B B_{m_j}(x) \]
where \( B_{m_j}(x) \) is the Brillouin
function of order \( m_j \) and \( x = (g \mu_B m_j H)/(k_B T) \).

5. (a) Find out an expression for the free energy of Fermi gas
under magnetic quantization.

(b) Explain the importance of radial distribution function
for amorphous materials and Born-Green-Yuon theory.