2015

M.Sc. Part–II Examination

PHYSICS

PAPER—VIII

Full Marks : 75

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate answer-scripts for Group-A and Group-B

Group–A

(Advanced Quantum Mechanics)

[Marks : 40]

Answer Q. No. *1, 2, 3* and *two* from the rest.

1. Answer any *five* bits : 2×5

(i) If the doublet splitting of the first excited state $^2P_{3/2} \rightarrow {}^2P_{1/2}$ of He^+ is 5.84 cm^{-1}, calculate the corresponding separation of Hydrogen atom.

(ii) Prove the optical theorem

$$\sigma_{total} = \frac{4\pi}{k} \text{Im}[f(0)]$$

where the symbols carry their usual meaning.

(iii) Indistinguishability of identical particles lead to symmetric or antisymmetric nature of wave-functions — Justify.

(iv) Consider a system of two identical particles each with spin $\frac{1}{2}$. The Hamiltonian of the system is given as

$$\hat{H} = A + B\hat{S}_1.\hat{S}_2 + C(\hat{S}_{1Z} + \hat{S}_{2Z})$$

Find the eigenvalues of the system.

(v) Express Dirac equation in covariant form and express the properties of γ matrices.

(vi) Show that

$$(\vec{\alpha}.\vec{A})(\vec{\alpha}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.^{d}(\vec{A} \times \vec{B})$$

(vii) Obtain eigenvalues of the operator

$$K = \frac{\beta(\vec{\sigma}.^{d}\vec{L} + \hbar)}{\hbar}$$

(viii) Derive the continuity equation for spin $\frac{1}{2}$ particles and explain the terms.

2. Answer any *two* bits : 3×2

(i) Find the scattering cross-section for scattering of a particle of mass m by the δ-function potential

$$V(\vec{r}) = g\,\delta(\vec{r}), \text{ where } g = \text{constant}.$$

(ii) Prove that $(\vec{\alpha}.\vec{B})(\vec{\alpha}.\vec{C}) = 4\vec{B}.\vec{C}$.

(iii) Show that $[\vec{S}, S_r] = i\hbar\frac{\vec{r} \times \vec{s}}{r}$

where $S_r = \frac{\vec{s}.\vec{r}}{r}$

3. Answer any *one* bit : 4

(i) Three non-interacting identical Fermions are in an infinite potential well denoted by

$$V(x) = 0 \quad \text{for} \quad 0 < x < a$$
$$= \infty \quad \text{for} \quad x < 0 \text{ and } x > a$$

What would be the ground state energy ?

(ii) Find the C.G. coefficients for $j_1 = j_2 = \frac{1}{2}$.

4. (a) Establish the expression of a plane wave in terms of spherical waves.

(b) In the partial wave analysis of scattering find the criterion for determining the significant number of spherical waves.

(c) Obtain an expression for the phase shift δ_0 for S-wave scattering by the potential

$$V(r) = \infty \quad \text{for} \quad 0 \le r \le a$$
$$= 0 \quad \text{for} \quad r > a$$

5+2+3

5. Obtain Dirac equation for a free particle and obtain its solution. Discuss various implication of negative energy states. 8+2

6. (a) If $V(r) = \frac{-ze^2}{2R}\left(3 - \frac{r^2}{R^2}\right)$ for $0 < r < R$

$$= \frac{-ze^2}{r} e^{-ar} \quad \text{for} \quad R < r < \infty$$

Show that form factor F(q) for high energy elastic scattering is given by

$$F(q) = \left(\frac{3}{q^2R^2}\right)\left(\frac{\text{Sin } qR}{qR} - \text{Cos } qR\right)$$

where q = momentum transfer wave vector. 5

(b) Describe Fermi-Thomas model of the atom and prove that

$$\frac{d^2\chi}{dx^2} = \frac{\chi^{3/2}}{x^{1/2}}.$$

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Group–B

(Statistical Mechanics)

[Marks : 35]

Answer Q. No. *1* and *two* from the rest.

1. Answer any *five* bits : 5×3

(a) A system of three cells such that $N_1 = 5, N_2 = 3, N_3 = 2$; $E_1 = 0, E_2 = 2, E_3 = 4$ joules per particle. If total no. of particles and energy are constant and $\delta N_3 = -2$ then find δN_1 and δN_2.

(b) A monoatomic crystalline solid comprises of N atoms. Out of which n atoms are in interstitial positions of the available interstitial sites are N_1, find the number of possible microstates.

(c) Find the partition function of quantum mechanical harmonic oscillator in two dimension.

(d) The density matrix of a system is given by

$$\rho = \begin{pmatrix} \theta & 0 \\ 0 & 1-\theta \end{pmatrix}$$

where $0 \leq \theta \leq 1$. Find the entropy. What is the entropy in a pure state ?

(e) Systems with finite number of microstates gives rise to concept of negative temperature — Explain.

(f) How Bragg William approximation predicts MFA ?

(g) Explain Landau energy levels and how degeneracy depends on magnetic field ?

(h) Explain the term 'symmetry breaking' for para-ferro transition.

2. (a) Prove that two-dimensional ideal B-E gas can not undergo B-E condensation.

(b) Write down the expression for free energy of FD gas under magnetic quantization. Prove that degree of degeneracy is given by

$$g = L_x L_y H \Big/ \left(\frac{hc}{e}\right)$$

for a two-dimensional system of dimension L_x, L_y with magnetic field H.

(c) Write down an expression for isothermal susceptibility according to G-L theory of phase transition and explain all the terms. 4+4+2

3. (a) Define long range and short range order parameter.

(b) Prove that temperature dependance of long range order parameter for Ising-spin system in a magnetic field $\vec{H} = \hat{e}_z H$ is given by

$$L(T) = \tanh \beta \, (J_e \gamma L + \mu_0 H)$$

where γ = no. of n.n. and other symbols have usual meanings.

(c) Also prove that

$$L = \frac{\sqrt{3(T_c - T)}}{T_c}$$

near transition temperature. 2+5+3

4. (a) Deduce an expression of B-E distribution function from grand partition function.

(b) Prove that the average dipole-magnetic moment $\langle \mu_z \rangle = g m_j \mu_B B_{mj}(x)$ where $B_{mj}(x)$ is the Brillouin function of order m_j and $x = (g\mu_B m_j H)/(k_B T)$. 5+5

5. (a) Find out an expression for the free energy of Fermi gas under magnetic quantization.

(b) Explain the importance of radial distribution function for amorphous materials and Born-Green-Yuon theory. 5+5

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