2015

M.Sc. Part-I Examination

PHYSICS

PAPER—II

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answerscripts for Gr. A & Gr. B.

Group—A

[Marks—50]

Answer *Q. No. 1* and *2* and any *two* from the rest.

1. Answer any *six* of the following : 6×2

(a) Show that particle velocity and group velocity are same for a relativistic particle.

(Turn Over)

(b For a Gaussian function.

$$\Psi(x) = A\, e^{\frac{-x^2}{2\sigma^2}} ; \ -\infty < x < \infty ,$$

Find the value of A.

(c) Show that the operator $|\psi\rangle\langle\psi|$ is a projection operator only when $|\psi\rangle$ is normalized.

(d) Prove that for a Hermitian operator, all of its eigenvalues are real and the eigenvector corresponding to different eigenvalues are orthogonal.

(e) Find the splitting of the $^1P \rightarrow {}^1S$ transition of an atom placed in a magnetic field along z-axis.

(f) If $[\hat{B}, \hat{A}] = 2\hat{A}$

show that $e^{\hat{B}}\,\hat{A}\,e^{-\hat{B}} = e^2\,\hat{A}$.

(g) How the state vector and the operator are constructed in Heisenberg picture ?

(h) If $\hat{H} = -\frac{d^2}{dx^2} + \hat{x}^2$

and $\psi(x) = Ax\, e^{-x^2/2}$ Find the eigen value of $\hat{H}$

(i) Evaluate

$$\int_2^9 \left(x^2 - 5x + 2\right) \delta\left[2\,(x-4)\right] dx .$$

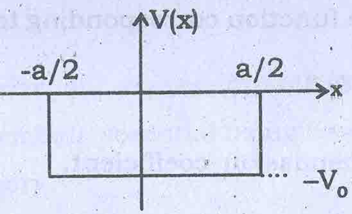
2. Answer any *three* bits : 3×4

(a) A particle is described by the wave function

$$\Psi(x) = \frac{1+ix}{1+ix^2}$$

Normalize $\psi(x)$. Where is the particle must likely to be found ?

(b)



There are only three bound states for a particle of mass m in a one dimensional potential well of the form shown in fig.

Show that the depth of the potential satisfies

$$\frac{2\pi^2\hbar^2}{ma^2} < V_o < \frac{8\pi^2\hbar^2}{ma^2}$$

(c) Show that the operator

$$\hat{A} = i\left(\hat{x}^2 + 1\right)\frac{d}{dx} + i\hat{x}$$ is Harmitian

(d) Show that linear momentum operator in $\left(\hat{I} + \frac{i}{\hbar}\,\delta\hat{p}_x\right)$ is a generator of infinitesimal spatial transformation.

3. (a) A particle of mass m is subjected to a delta potential

$$V(x) = \infty,\ x \le o$$
$$= V_o\,\delta(x-a),\ x > o \quad \text{where} \quad V_o \rangle o.$$

(i) Find the wave function corresponding to the cases $o \langle x \langle a$ and $x \rangle a$.

(ii) Find the transmission coefficient.

(b) The wave function for Hydrogen atom is

$$\psi(IS) = \frac{1}{\sqrt{\pi a_o^3}}\, e^{-r/a_o}$$

Where $a_o = \frac{\hbar^2}{me^2}$. Find the expectation value of the potential energy $V(r) = \frac{-e^2}{r}$. 4+4+5

4. (a) If $\left[\hat{A}, \hat{B}\right] = i\,\hat{c}$; $\hat{A}, \hat{B}$ are Hermitian.

Then prove that $\left(\Delta\hat{A}\right)\left(\Delta\hat{B}\right) \ge \frac{\langle\hat{c}\rangle}{2}$.

(b) A particle of mass m in the potential

$$V(x) = \infty \text{ for } x \le o$$
$$= \frac{1}{2} m w^2 x^2 \quad \text{for} \quad x \rangle o.$$

Find the ground state energy for this particle using (i) Variational method, (ii) WKB approx, method. 5+4+4

5. (a) Derive an expression for transition probability for Harmonic potential using time dependent perturbation theory.

(b) A particle which is initially $(t = 0)$ is in the ground state of an infinite, one dimensional potential box with the walls at $x = 0$ and $x = a$, is subjected for $0 \le t \le \infty$ to a perturbation $\hat{V}(t) = x^2\, \bar{e}^{\,t/\tau}$ calculate to first order the probability of finding the particle in its first excited state for $t \ge 0$. 8+5

Group—B

[Marks—25]

Answer *Q. No. 1* and any *two* from the rest

1. Answer any *three* : 3×3

(a) Assuming a simple harmonic oscillator find the expression of wave number ω_{ose}.

(b) The first line in the rotation spectrum of carbon monoxide as $3{\cdot}84235\,cm^{-1}$. Find the moment of Inertia.

(c) What is Born oppenheimer approximation for electronic spectra of diatomic molecule. How a small change in electronic energy is related to change in vibrational energy.

(d) Diatomic molecules such as HCl, HBr, CO, HF will show a rotational spectrum where as N_2, O_2, H_2 will not why ? Will the molecule $^{17}O - {}^{16}O$ show a rotational spectrum?

(e) What is centrifugal distortion ? Explain the effect of centrifugal distortion on the moment of inertia and energy of a diatomic molecule.

(f) Convert the following spectroscopic quantities as indicated :

(i) $40{\cdot}97 \times 10^{-23}$ J to cm^{-1}

(ii) 5 μm to cm^{-1}

(iii) $41{\cdot}22\,cm^{-1}$ to J

(g) Give the working principle of Rubby laser.

2. What is meant by Vibrational Coarse Structure ? Explain how the intensity of vibrational lines vary in progression ? 4+4

3. Assuming the diatomic Vibrating rotator (following Born oppenheimer approximation) find the transition between the rotational vibrational energy levels. 8

4. A microwave spectrometer capable to operating only between 60 and 90 cm^{-1} was used to observe the rotational spectra of HI. Absorptions were measured as follows. 64·275, 77·130, 89·985.

Find B, I and r and determine the J values between which transitions occur for the first line listed above

M_H $1{\cdot}673 \times 10^{-27}$ Kg ; $M_I = 210{\cdot}7 \times 10^{-27}$;

$h = 6{\cdot}626 \times 10^{-34}$ JS. 1+2+2+3

5. The rotational constant for $H^{35}Cl$ is observed to be $10{\cdot}5909\ cm^{-1}$. What are the values of B for $H^{37}Cl$ and for ${}^{2}D^{35}Cl$?

$M_H = 1{\cdot}637 \times 10^{-27}$ Kg.

$M_{35Cl} = 58{\cdot}06 \times 10^{-27}$ Kg.

$M_D = 3{\cdot}344 \times 10^{-27}$ Kg.

The first line in the rotational spectrum of CO has a frequency of $3{\cdot}8424\ cm^{-1}$. Calculate the rotational constant and bondlength of C–0 in carbon monoxide.

$M_c = 12$ gm/mol $M_o = 15{\cdot}9949$ gm/mol.
