

2015

**M.Sc. Part-I Examination**

**PHYSICS**

**PAPER—I**

*Full Marks : 75*

*Time : 3 Hours*

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**Use separate Answerscripts for Gr. A & Gr. B.**

**Group—A**

[Marks : 30]

1. Answer any *four* of the following : 4×2

- (a) In Rutherford's  $\alpha$ -particles scattering experiment  $10^{-3}$   $\alpha$ -particles are scattered at an angle  $4^\circ$ , calculate the number of  $\alpha$ -particles, scattered at an angle  $15^\circ$ .

(Turn Over)

- (b) The potential energy of a particle is given as

$$v(y) = y^4 - 4y^3 - 8y^2 + 48y$$

Find out the stable and unstable equilibrium position of the particle.

- (c) Find the Lagrangian's equation of motion for a electrical circuit consisting an inductor  $L$  and capacitor  $C$ . The capacitor is charged to  $q$  coulomb and current flowing in the circuit is  $i$  amp.

- (d) Prove that total energy ( $E$ ) of a particle in a rotating frame is related to  $E_0$  in the fixed frame by the rotation  $E = E_0 - \bar{\omega} \cdot \bar{L}$ , where  $\bar{\omega}$  is the angular velocity of the rotating frame and  $\bar{L}$  is the angular momentum of the particle in the same frame.

- (e) What do you mean by Fast Top and Sleeping Top? Write the difference between them.

- (f) When a torque ( $\bar{\tau}$ ) is applied to a body, then prove

$$\text{that } \bar{\omega} \cdot \bar{\tau} = \frac{dT}{dt}$$

where  $\bar{\omega}$  is the angular velocity of the body and  $T$  is the kinetic energy of the body.

- (g) Find out the equation of motion of one dimensional harmonic oscillator using Hamilton's Principle.

2. Answer any two of the following : 2×3

- (a) The Lagrangian of a system is given as

$$L = \frac{1}{2} \alpha_1 \dot{\phi}^2 + \frac{1}{2} \alpha_2 \dot{\psi}^2 + \alpha_3 \dot{\phi} \dot{\psi} - \frac{1}{2} \alpha_4 \phi^2 - \frac{1}{2} \alpha_5 \psi^2$$

where  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\alpha_5$  are constants. Find out the expressions of  $P_\phi, P_\psi, H$  and Hamilton's canonical function.

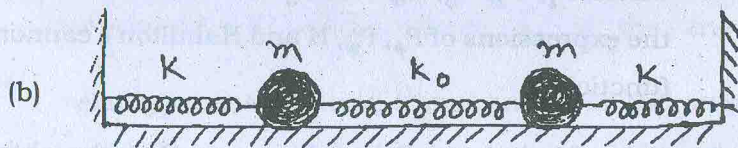
- (b) Using Variational Principle, prove that the shortest distance between any two points in a plane is a straight line.

- (c) Deduce the expression of normal mode frequencies of a linear triatomic molecule of the type  $AB_2$ . Consider only the vibrational motion along the line joining the atoms.

(d) Deduce Lagrange's equation of motion from Hamilton's principle for a Conservative Systems.

3. Answer any *two* of the followings : 2×8

- (a) (i) State principle of least action. Deduce Jacobi's form of the principle of least action.
- (ii) If KE of a system is conserved, then show that out of all paths between two points, the system moves along the particular path for which the time of transit is an extreme. (2+3)+3



Find out the frequencies of normal modes of the small oscillation of the above spring mass system.

Consider only longitudinal oscillation. Here  $K$  and  $K_0$  are the Spring Constants.

What do you mean by Symmetric and anti-Symmetric mode in this case?

What will happen if the spring between the masses

of spring constant  $K_0$  is replaced by:

- (i) an another spring of spring constant  $K$  ;
- (ii) a rod of fixed length and of negligible mass.

4+2+2

- (c) (i) Deduce the solution of one dimensional harmonic oscillator problem by Hamilton-Jacobi method.
- (ii) Prove that for Harmonic Oscillator, the Hamilton's Principal function is the time integral of Lagrangian. 5+3

### Group—B

[Marks : 45]

Answer Q. No. 1 and any *three* from the rest.

1. Answer any *three* from the following : 3×3

- (a) Explain what is meant by center of symmetry and mirror plane.
- (b) Cu has Fcc structure and atomic radius is 0.1278 nm. Calculate the interplanar spacing for 110 plane.

- (c) Ignoring the difference between transverse and longitudinal velocity show that Debye Temperature is

$$\text{given by } \theta_D = \frac{h\lambda}{2\pi k} \left[ 6\pi^2 \frac{N}{V} \right]^{\frac{1}{3}}$$

- (d) Find the average energy of the electron at absolute zero assuming 3-dimensional Fermi gas in a metal.

- (e) Prove that effective mass can be expressed as

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

What is meant by negative effective mass?

- (f) What is meant by Single crystal and Poly crystal. How can you identify them by Lave Diffraction Method?

2. (a) Derive Lave equations considering the scattering of X-rays from a small crystal.

- (b) Find the condition for systematic absence in a B.C.C. crystal. 10+2

3. (a) Derive the dispersion relation assuming vibration in an one-dimensional monoatomic lattice.

- (b) What is Van-Hove Singularity?

- (c) Sound velocities in solids are of the order of  $3 \times 10^3 \text{ m s}^{-1}$ . Interatomic distances in solid is  $3 \times 10^{-10} \text{ m}$ . Estimate the order of magnitude of cut off frequency assuming a linear lattice. 7+2+3

4. (a) Prove that average energy of electron in a metal increases as temperature increases.

- (b) Find an expression of electronic specific heat.

- (c) Explain what is Hall effect? 7+2+3

5. (a) Find the energy of electron in a solid according to Tight Binding Approximation.

- (b) Assuming Tight Binding Approximation find the Bandwidth of a Simple Cubic Lattice along [111]. 9+3

6. (a) Define polarization in a dielectric & hence derive  $P = \frac{(\epsilon_s - 1)E}{4\pi}$  where  $\epsilon_s$  is the static dielectric constant.

- (b) Suppose a solid contains N atom per  $\text{cm}^3$ , each atom

containing  $Z$  electron, the diamagnetic susceptibility may be expressed as,

$$\chi_{\text{dia}} = -NZ \left( \frac{e^2}{6mc^2} \right) \langle r^2 \rangle$$

where  $\langle r^2 \rangle$  represents mean square distance of the electrons from the nucleus.

(c) What is Meissner Effect ?

4+6+2

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