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On T_1 Space in L-Topological Spaces

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ABSTRACT

In this paper, we introduce eight notions of T_1 space in L-topological spaces. We established some relation among them. Also, we prove that all of these definitions satisfy "good extension" property. Finally, we prove that all of these notions are hereditary, productive and projective. Moreover, we observe that all concepts are preserved under one-one, onto and continuous mapping.

Keywords: L-Fuzzy Set, Fuzzy topological space, L-Topological spaces, Hereditary.

1. Preliminary and Introduction

The fundamental concept of fuzzy set introduced by Zadeh [23] provided a natural foundation for building new branches. In 1968 Chang [2] introduced the concept of fuzzy topological spaces and there after many fuzzy topologists, have contributed various forms of separation axioms to the theory of fuzzy topological spaces. The L-fuzzy topological spaces were defined by Kubiak [9], Sostake [20,21] and their co-workers.

Through this paper, X will be a nonempty set, I = [0, 1] and L be a complete distributive lattice with 0 and 1. The class of all L-fuzzy sets on a universe X will be denoted by L^X and L-fuzzy sets on X will be denoted by u, v, w, etc. Crisp subset of X will be denoted by capital letter A, B, C etc. L-fuzzy singleton will be denoted by x_r, y_r, z_r. The class of all fuzzy singletons in X is denoted by S(X). For every $x_t \in S(X)$ and $v \in L^X$, we write $x_r \in v$ iff $r \leq v(x)$, also by $\alpha(x) = \alpha$, $\forall\, x \in X$ and $\alpha \in I$, we mean the constant mapping on X with value α and 1_A denoted the characteristic mapping of $A \subseteq X$.

Definition 1.1. [23] Let X be a non-empty set and I = [0, 1]. A fuzzy set in X is a function $u: X \to I$ which assign to each element $x \in X$, a degree of membership, $u(x) \in I$.

Definition 1.2. [4] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An L-fuzzy set in X is a function $\alpha: X \to L$ which assign to each element $x \in X$, a degree of membership, $\alpha(x) \in L$.

Definition 1.3. [4] Let α be an L-fuzzy set in X.Then $1 - \alpha = \alpha'$ is called the complement of α in X (Zadeh 1965).

Definition 1.4. [4] If $r \in L$ and α is an L-fuzzy sets in X defined by $\alpha(x) = r$, $\forall\, x \in X$ then we refer to α as a constant L-fuzzy sets and denoted it by r itself.

In particular, we have the constant L-fuzzy sets 0 and 1.

Definition 1.5. [12] An L-fuzzy point p in X is a special L-fuzzy sets with membership function $p(x) = r \;\; if \;\; x = x_0$
$p(x) = 0 \;\; if \;\; x \neq x_0$ where $r \in L$.

Definition 1.6. [12] An L-fuzzy point p is said to belong to an L-fuzzy set α in $X(p \in \alpha)$ if and only if $p(x) < \alpha(x)$ and $p(y) \leq \alpha(y)$. That is $x_r \in \alpha \; implies \;\; r < \alpha(x)$.

Definition 1.7. [2] Let $I = [0,1]$, X be a non-empty set and I^X be the collection of all mappings from X into I , i. e. the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions: $(i) 1, 0 \in t$ (ii) if $u_i \in t$ for each $i \in \Delta$ then $\bigcup_{i \in \Delta} u_i \in t$ (iii) if $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$. The pair (X, t) is called a fuzzy topological space (fts, in short) and the members of t are called t-open (or simply open) fuzzy sets. A fuzzy set v is called a t-closed (or simply closed) fuzzy set if $1 - v \in t$.

Definition 1.8. [9] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. Suppose that τ be the sub collection of all mappings from X to $L \; i.e., \tau \subseteq L^X$.Then τ is called L-topology on X if it satisfies the following conditions:

- (i) $0^*, 1^* \in \tau$
- (ii) If $u_1, u_2 \in \tau$ then $u_1 \cap u_2 \in \tau$
- (iii) If $u_i \in \tau$ for each $i \in \Delta$ then $\cup_{i \in \Delta} u_i \in \tau$.

Then the pair (X, τ) is called a L-topological space (lts, for short) and the members of τ are called open L-fuzzy sets. An L-fuzzy sets v is called a closed L-fuzzy set if $1 - v \in \tau$.

Definition 1.9. [9] Let λ be an L-fuzzy set in lts (X, τ). Then the closure of λ is denoted by $\bar{\lambda}$ and defined as $\bar{\lambda} = \cap \{\mu : \lambda \subseteq \mu , \mu \in \tau^c\}$.
The interior of λ written λ^0 is defined by $\lambda^0 = \cup \{\mu : \mu \subseteq \lambda , \mu \in \tau\}$.

Definition 1.10. [9] An L-fuzzy singleton in X is an L-fuzzy set in X which is zero everywhere except at one point say x, where it takes a value say r with $0 < r \leq 1$ and $r \in L$. We denote it by x_r and $x_r \in \alpha$ iff $r \leq \alpha(x)$.

Definition 1.11. [9] An L-fuzzy singleton x_r is said to be quasi-coincident (q-coincident, in short) with an L-fuzzy set α in X, denoted by $x_r q \alpha$ iff $r + \alpha(x) > 1$. Similarly, an L-fuzzy set α in X is said to be q-coincident with an L-fuzzy set β in X, denoted by $\alpha q \beta$ if and only if $\alpha(x) + \beta(x) > 1$ for some $x \in X$. Therefore $\alpha \bar{q}\, \beta$

iff $\alpha(x) + \beta(x) \leq 1$ for all $x \in X$, where $\alpha\bar{q}\,\beta$ denote an L-fuzzy set α in X is said to be not q-coincident with an L-fuzzy set β in X.

Definition 1.12. [2] Let X and Y are two sets and $f: X \rightarrow Y$is a function. For a fuzzy subset u in X, we define a fuzzy subset v in Y by
$v(y) = sup\{u(x)\}$if$f^{-1}[\{y\}] \neq \emptyset, x \in X$

$v(y) = 0$ if $f^{-1}[\{y\}] = \emptyset, x \in X$.

Definition 1.13. [14] Let f be a real-valued function on an L-topological space. If $\{x: f(x) > \alpha\}$ is open for every real α, then f is called lower-semi continuous function (lsc, in short).

Definition 1.14. [15] Let (X, τ) and (Y, s) be two L-topological space and f be a mapping from (X, τ) into (Y, s) $i.e. f: (X, \tau) \rightarrow (Y, s)$. Then f is called-

(i) Continuous iff for each open L-fuzzy set $u \in s \Longrightarrow f^{-1}(u) \in \tau$.
(ii) Open iff $f(\mu) \in s$ for each open L-fuzzy set $\mu \in \tau$.
(iii) Closed iff $f(\lambda)$ is s-closed for each $\lambda \in \tau^c$ where τ^c is closed L-fuzzy set in X.
(iv) Homeomorphism iff f is bijective and both f and f^{-1} are continuous.

Definition 1.15. [6] Let X be a nonempty set and T be a topology on X. Let $\tau = \omega(T)$ be the set of all lower semi continuous (lsc) functions from (X, T) to L (with usual topology). Thus $\omega(T) = \{u \in L^X: u^{-1}(\alpha, 1] \in T\}$ for each $\in L$. It can be shown that $\omega(T)$ is a L-topology on X. Let "P" be the property of a topological space (X, T) and LP be its L-topological analogue. Then LP is called a "good extension" of P "if the statement (X, T) has P iff $(X, \omega(T))$ has LP" holds good for every topological space (X, T).

Definition 1.16. [24] Let $\{(X_i, \tau_i): i \in \Delta\}$ be a family of L-topological space. Then the space $(\Pi X_i, \Pi\tau_i)$ is called the product lts of the family $\{(X_i, \tau_i): i \in \Delta\}$ where $\Pi\tau_i$ denote the usual product L-topologies of the families $\{\tau_i: i \in \Delta\}$ of L-topologies on X.

2. T_1-property in L-topological spaces

Here, we define the following definitions of T_1-property in L-topological spaces.

Definition 2.1. An lts(X, τ) is called-

(a) $L - T_1(i)$ if $\forall\, x, y \in X, x \neq y$ then $\exists\, u, v \in \tau$ such that $u(x) \neq u(y)$ and $v(x) \neq v(y)$.
(b) $L - T_1(ii)$ if $\forall\, x, y \in X, x \neq y$ then $\exists\, u, v \in \tau$ such that $u(x) = 1, u(y) = 0$ and $v(x) = 0$, $v(y) = 1$.
(c) $L - T_1(iii)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ then $\exists\, u, v \in \tau$ such that $x_r \in u, y_s \notin u$ and $x_r \notin v, y_s \in v$.

(d) $L - T_1(iv)$ if for all pairs of distinct L-fuzzy singletons $x_r, y_s \in S(X)$ with $x_r \bar{q} y_s$ then $\exists\, u, v \in \tau$ such that $x_r \subseteq u,\ y_s \bar{q} u$ and $y_s \subseteq v, x_r \bar{q} v$.

(e) $L - T_1(v)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ then $\exists\, u, v \in \tau$ such that $x_r \in u, u \bar{q} y_s$ and $y_s \in v, v \bar{q} x_r$.

(f) $L - T_1(vi)$if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ then $\exists\, u, v \in \tau$ such that $x_r \in u, y_s \cap u = 0$ and $y_s \in v, x_r \cap v = 0$.

(g) $L - T_1(vii)$if $\forall\, x, y \in X, x \neq y$ then $\exists\, u, v \in \tau$ such that $u(x) > 0, u(y) = 0$ and $v(x) = 0,\ v(y) > 0$.

(h) $L - T_1(viii)$if $\forall\, x, y \in X, x \neq y$ then $\exists\, u, v \in \tau$ such that $u(x) > u(y)$ and $v(y) > v(x)$.

Here, we established a comparison of the definitions $L - T_1(ii), L - T_1(iii), L - T_1(iv)$, $L - T_1(v),\ L - T_1(vi), L - T_1(vii), L - T_1(viii)$ with $L - T_1(i)$is given below:

Theorem 2.2. Let (X, τ) be an lts. Then we have the following implications:

$$L - T_1(viii) \to L - T_1(vii) \to L - T_1(ii) \to L - T_1(i)$$

$L - T_1(v) \to L - T_1(i)$, $L - T_1(vi) \to L - T_1(i)$, $L - T_1(iii) \to L - T_1(i)$, $L - T_1(iv) \to L - T_1(i)$

The reverse implications are not true in general.

Proof: $L - T_1(ii) \Rightarrow L - T_1(i), L - T_1(iii) \Rightarrow L - T_1(i)$ and $L - T_1(iv) \Rightarrow L - T_1(i)$ can be proved easily. Now $L - T_1(v) \Rightarrow L - T_1(i)$ since $L - T_1(v) \Leftrightarrow L - T_1(iii)$.

$L - T_1(vi) \Rightarrow L - T_1(i)$, since $L - T_1(vi) \Rightarrow L - T_1(v)$. $L - T_1(vii)$ and $L - T_1(viii) \Rightarrow L - T_1(i)$ since $L - T_1(viii) \Rightarrow L - T_1(vii)$ and $L - T_1(vii) \Rightarrow L - T_1(ii)$.

None of the reverse implications are true; it can be seen through the following:

Example 2.2.1. Let $X = \{x, y\}$, τ be the L-topology on X generated by $\{\alpha : \alpha \in L\} \cup \{u, v\}$ where$u(x) = 0.5,\ u(y) = 0.6$ and $v(x) = 0.7, v(y) = 0.4$ and
$L = \{0, 0.05, 0.1, 0.15, \ldots \ldots \ldots 0.95, 1\}$.

Proof: $L - T_1(i) \nRightarrow L - T_1(ii)$: Here the lts$(X, \tau)$ is clearly$L - T_1(i)$ but it is not $L - T_1(ii)$. Since there is no none empty L-fuzzy set in τ which takes zero value at x or y.
$L - T_1(i) \nRightarrow L - T_1(iii)$: For if we take the distinct L-fuzzy points $x_{3/5}, y_{1/2} \in S(X)$,then there does not exist $u, v \in \tau$ such that $x_{3/5} \in u, y_{1/2} \notin u$ and $x_{3/5} \notin v, y_{1/2} \in v$.
$L - T_1(i) \nRightarrow L - T_1(iv)$: As for the distinct L-fuzzy singletons x_1, y_1 in τ there does not exist $u, v \in \tau$ such that $x_1 \subseteq u, y_1 \bar{q} u$ and $y_1 \subseteq v, x_1 \bar{q} v$.
$L - T_1(i) \nRightarrow L - T_1(v)$: This follows automatically from the fact that $L - T_1(v) \Leftrightarrow L - T_1(iii)$and it has already been shown that $L - T_1(i) \nRightarrow L - T_1(iii)$.
$L - T_1(i) \nRightarrow L - T_1(vi)$: Since for any two distinct L-fuzzy points $x_{3/5}, y_{1/2}$ in $S(X)$, then there does not exist $u, v \in \tau$ which is disjoint with $x_{3/5}$ and $y_{1/2}$.
$L - T_1(i) \nRightarrow L - T_1(vii)$ and $L - T_1(i) \nRightarrow L - T_1(viii)$: It is obvious because
$L - T_1(vii) \Rightarrow L - T_1(ii)$ and $L - T_1(viii) \Rightarrow L - T_1(ii)$ and it has already been shown that $L - T_1(i) \nRightarrow L - T_1(ii)$.

3. "Good extension", hereditary, productive and projective properties in l-topology
Here, we show that all these definitions $L-T_1(i)$,$L-T_1(ii)$, $L-T_1(iii)$,

$L-T_1(iv)$, $L-T_1(v)$, $L-T_1(vi)$, $L-T_1(vii)$ and $L-T_1(viii)$ are 'good extensions 'of T_1- property, as shown below:

Theorem 3.1. Let (X,T) be a topological space. Then (X,T) is T_1 iff $(X,\omega(T))$ is $L-T_1(i)$.
Proof: Let (X,T) be T_1.Choose $x,y\in X$with $x\neq y$.Then $\exists\, U,V\in T$ such that $x\in U, y\notin U$ and $y\in V, x\notin V$. Now consider the lower semi continuous functions $1_U, 1_V$. Then $1_U, 1_V\in\omega(T)$ with $1_U(x)=1$, $1_U(y)=0$ and $1_V(x)=0, 1_V(y)=1$ and so that $1_U(x)\neq 1_U(y)$ and $1_V(x)\neq 1_V(y)$. Thus $(X,\omega(T))$ is $L-T_1(i)$.
Conversely, let $(X,\omega(T))$ be $L-T_1(i)$. To show that (X,T) is T_1. Choose $x,y\in X$ with $x\neq y$. Then $\exists\, u,v\in\omega(T)$ such that $u(x)\neq u(y)$ and $v(x)\neq v(y)$. Let $u(x)<u(y)$ and $v(y)<v(x)$. Choose r and s such that $u(x)<r<u(y)$ and $v(y)<s<v(x)$ and consider $u^{-1}(r,1]$ and $v^{-1}(s,1]$. Then $u^{-1}(r,1], v^{-1}(s,1]\in T$ and is $x\notin u^{-1}(r,1]$,$y\in u^{-1}(r,1]$ and $x\in v^{-1}(s,1]$, $y\notin v^{-1}(s,1]$. Hence (X,T) is T_1.
Similarly we can show that $L-T_1(ii), L-T_1(iii), L-T_1(iv)$, $L-T_1(v)$, $L-T_1(vi)$, $L-T_1(vii), L-T_1(viii)$are also hold 'good extension' property.

Theorem 3.2. Let (X,τ) be an lts, $A\subseteq X$ and $\tau_A=\{u|A: u\in\tau\}$, then

(a) (X,τ) is $L-T_1(i)\Rightarrow(A,\tau_A)$ is $L-T_1(i)$.
(b) (X,τ) is $L-T_1(ii)\Rightarrow(A,\tau_A)$ is $L-T_1(ii)$.
(c) (X,τ) is $L-T_1(iii)\Rightarrow(A,\tau_A)$ is $L-T_1(iii)$.
(d) (X,τ) is $L-T_1(iv)\Rightarrow(A,\tau_A)$ is $L-T_1(iv)$.
(e) (X,τ) is $L-T_1(v)\Rightarrow(A,\tau_A)$ is $L-T_1(v)$.
(f) (X,τ) is $L-T_1(vi)\Rightarrow(A,\tau_A)$ is $L-T_1(vi)$.
(g) (X,τ) is $L-T_1(vii)\Rightarrow(A,\tau_A)$ is $L-T_1(vii)$.
(h) (X,τ) is $L-T_1(viii)\Rightarrow(A,\tau_A)$ is $L-T_1(viii)$.

Proof: We prove only (b). Suppose $(X,\tau)$is L-topological space and $L-T_1(ii)$.We shall prove (A,τ_A) is $L-T_1(ii)$. Let $x,y\in A$ with $x\neq y$, then $x,y\in X$ with $x\neq y$as $A\subseteq X$. Since $(X,\tau)$is $L-T_1(ii)$, $\exists\, u,v\in\tau$ such that $u(x)=1, u(y)=0$ and $v(x)=0$, $v(y)=1$. For $A\subseteq X$ we find $u|A, v|A\in\tau_A$ and $u|A(x)=1, u|A(y)=0$ and $v|A(x)=0, v|A(y)=1$ as $x,y\in A$. Hence it is clear that the subspace (A,τ_A) is $L-T_1(ii)$. Similarly, (a), (c), (d), (e), (f), (g), (h) can be proved.
i.e. L-T_1(j), for j = i, ii, iii,… (viii). Satisfy hereditary property.

Theorem 3.3. Given $\{(X_i,\tau_i): i\in\Lambda\}$ be a family of L-topological space. Then the product of L-topological space $(\Pi X_i,\Pi\tau_i)$ is $L-T_1(j)$ iff each coordinate space (X_i,τ_i) is $L-T_1(j)$ where$j=i,ii,iii,iv,v,vi,vii,viii$.
Proof: Let each coordinate space $\{(X_i,\tau_i): i\in\Lambda\}$ be $L-T_1(ii)$. Then we show that the product space is $L-T_1(ii)$.Suppose $x,y\in X$ with $x\neq y$, again suppose $x=\Pi x_i, y=\Pi y_i$ then $x_j\neq y_j$ for some $j\in\Lambda$. Now consider $x_j, y_j\in X_j$. Since $(X_j,\tau_j)$is $L-T_1(ii)$, $\exists\, u_j, v_j\in\tau_j$ such that $u_j(x_j)=1, u_j(y_j)=0$ and $v_j(x_j)=0, v_j(y_j)=1$. Now take

$u = \Pi u'_j, v = \Pi v'_j$ where $u'_j = u_j, v'_j = v_j$ and $u_i = v_i = 1$for $i \neq j$. Then $u, v \in \Pi\tau_i$ such that $u(x) = 1, u(y) = 0$ and $v(x) = 0, v(y) = 1$. Hence the product L-topological space $(\Pi X_i, \Pi\tau_i)$ is $L - T_1(ii)$.

Conversely, let the product L-topological space $(\Pi X_i, \Pi\tau_i)$is $L - T_1(ii)$.Take any coordinate space(X_j, τ_j), choose $x_j, y_j \in X_j, x_j \neq y_j$. Now construct $x, y \in X$ such that $x = \Pi x'_i, y = \Pi y'_i$where $x'_i = y'_i$ for $i \neq j$ and $x'_j = x_j, y'_j = y_j$. Then $x \neq y$ and using the product space $L - T_1(ii) \exists\, u, v \in \Pi\tau_i$ such that $u(x) = 1, u(y) = 0$ and $v(x) = 0$, $v(y) = 1$. Now choose any L-fuzzy point x_r in u. Then $\exists$ a basic open L-fuzzy set $\Pi u_j^r \in \Pi\tau_j$ such that $x_r \in \Pi u_j^r \subseteq u$ which implies that $r < \Pi u_j^r(x)$ or that $r < inf_j u_j^r(x'_j)$ and hence $r < \Pi u_j^r(x'_j) \forall\, j \in \Lambda \ldots\ldots (i)$ and $u(y) = 0 \Rightarrow \Pi u_j(y) = 0 \ldots\ldots (ii)$. Similarly, corresponding to a fuzzy point $y_s \in v$ there exists a basic open L-fuzzy set $\Pi v_j^s \in \Pi\tau_j$ that $y_s \in \Pi v_j^s \subseteq v$ which implies that $s < v_j^s(j) \forall\, j \in \Lambda \ldots\ldots (iii)$ and $v_j^s(y) = 0 \ldots\ldots (iv)$. Further, $\Pi u_j^r(y) = 0 \Rightarrow u_i^r(y_i) = 0$, since for $j \neq i, x'_j = y'_j$ and hence from (i),$u_j^r(y_j) = u_j^r(x_j) > r$. Similarly, $\Pi v_j^s(x) = 0 \Rightarrow v_i^s(x_i) = 0$ using(iii). Thus we have $u_i^r(x_i) > r, u_i^r(y_i) = 0$ and $v_i^s(y_i) > s, v_i^s(x_i) = 0$. Now consider $sup_r u_i^r = u_i, sup_s v_i^s = v_i \in \tau_i$ then $u_i(x_i) = 1, u_i(y_i) = 0$ and $v_i(x_i) = 0, v_i(y_i) = 1$ showing that (X_i, τ_i) is $L - T_1(ii)$.

Moreover one can verify that

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(i) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(i)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(iii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(iii)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(iv) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(iv)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(v) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(v)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(vi) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(vi)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(vii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(vii)$

$(X_i, \tau_i), i \in \Lambda$ is $L - T_1(viii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L - T_1(viii)$.

Hence we see that $L - T_1(i), L - T_1(ii), L - T_1(iii), L - T_1(iv), L - T_1(v)$,

$L - T_1(vi), L - T_1(vii), L - T_1(viii)$Properties are productive and projective.

4. Mapping in L-topological spaces

We show that $L - T_1(j)$ property is preserved under one-one, onto and continuous mapping for $j = i, ii, iii, iv, v, vi, vii, viii$.

Theorem 4.1. Let (X, τ) and (Y, s) be two L-topological spaces and $f: (X, \tau) \to (Y, s)$ be one-one, onto and L-open map, then-

(a) (X, τ) is $L - T_1(i) \Rightarrow (Y, s)$ is $L - T_1(i)$.
(b) (X, τ) is $L - T_1(ii) \Rightarrow (Y, s)$ is $L - T_1(ii)$.
(c) (X, τ) is $L - T_1(iii) \Rightarrow (Y, s)$ is $L - T_1(iii)$.

(d) (X,τ) is $L-T_1(iv) \Rightarrow (Y,s)$ is $L-T_1(iv)$.
(e) (X,τ) is $L-T_1(v) \Rightarrow (Y,s)$ is $L-T_1(v)$.
(f) (X,τ) is $L-T_1(vi) \Rightarrow (Y,s)$ is $L-T_1(vi)$.
(g) (X,τ) is $L-T_1(vii) \Rightarrow (Y,s)$ is $L-T_1(vii)$.
(h) (X,τ) is $L-T_1(viii) \Rightarrow (Y,s)$ is $L-T_1(viii)$.

Proof: Suppose (X,τ) is $L-T_1(ii)$.We shall prove that (Y,s) is $L-T_1(ii)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto,$\exists\, x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$ and $x_1 \neq x_2$ as f is one-one. Again since (X,τ) is $L-T_1(ii) \exists\, u, v \in \tau$ such that $u(x_1) = 1$, $u(x_2) = 0$ and $v(x_1) = 0, v(x_2) = 1$.

Now, $f(u)(y_1) = \{sup u(x_1): f(x_1) = y_1\} = 1$

$f(u)(y_2) = \{sup u(x_2): f(x_2) = y_2\} = 0$ and

$f(v)(y_1) = \{sup v(x_1): f(x_1) = y_1\} = 0$

$f(v)(y_2) = \{sup v(x_2): f(x_2) = y_2\} = 1$.

Since f is L-open, $f(u), f(v) \in s$. Now it is clear that $\exists\, f(u), f(v) \in s$ such that $f(u)(y_1) = 1, f(u)(y_2) = 0$ and $f(v)(y_1) = 0, f(v)(y_2) = 1$. Hence it is clear that the L-topological space (Y,s) is $L-T_1(ii)$.Similarly (a), (c), (d), (e),(f), (g), (h) can be proved.

Theorem 4.2. Let (X,τ) and (Y,s) be two L-topological spaces and $f: (X,\tau) \to (Y,s)$ be L-continuous and one-one map, then-

(a) (Y,s) is $L-T_1(i) \Rightarrow (X,\tau)$ is $L-T_1(i)$.
(b) (Y,s) is $L-T_1(ii) \Rightarrow (X,\tau)$ is $L-T_1(ii)$.
(c) (Y,s) is $L-T_1(iii) \Rightarrow (X,\tau)$ is $L-T_1(iii)$.
(d) (Y,s) is $L-T_1(iv) \Rightarrow (X,\tau)$ is $L-T_1(iv)$.
(e) (Y,s) is $L-T_1(v) \Rightarrow (X,\tau)$ is $L-T_1(v)$.
(f) (Y,s) is $L-T_1(vi) \Rightarrow (X,\tau)$ is $L-T_1(vi)$.
(g) (Y,s) is $L-T_1(vii) \Rightarrow (X,\tau)$ is $L-T_1(vii)$.
(h) (Y,s) is $L-T_1(viii) \Rightarrow (X,\tau)$ is $L-T_1(viii)$.

Proof: Suppose (Y,s) is $L-T_1(ii)$.We shall prove that $(X,\tau)$is $L-T_1(ii)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2, \Rightarrow f(x_1) \neq f(x_2)$ as f is one-one. Since (Y,s) is $L-T_1(ii)$, $\exists\, u, v \in s$ such that $u(f(x_1)) = 1, u(f(x_2)) = 0$ and$v(f(x_1)) = 0, v(f(x_2)) = 1$.This implies that $f^{-1}(u)(x_1) = 1, f^{-1}(u)(x_2) = 0$ and $f^{-1}(v)(x_1) = 0, f^{-1}(v)(x_2) = 1$ and hence $f^{-1}(u), f^{-1}(v) \in \tau$ as f is L-continuous and $u, v \in s$ .Now it is clear that $f^{-1}(u), f^{-1}(v) \in \tau$ such that $f^{-1}(u)(x_1) = 1, f^{-1}(u)(x_2) = 0$ and $f^{-1}(v)(x_1) = 0, f^{-1}(v)(x_2) = 1$. Hence the L-topological space (X,τ) is $-T_1(ii)$. Similarly (a), (c), (d), (e), (f), (g), (h) can be proved.

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