Ph.D Synopsis

Some Aspects of Tunneling in Semiconductor Superlattices

Submitted by

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Synopsis

The phenomenon of tunneling of electrons through multibarrier systems (MBS) constitutes an important dimension of condensed matter of physics. The study of this phenomenon has been drawing intense attention during last thirty years [1-21]. The quickly progressing technology of semiconductor quantum structures depends on reliable predictive theoretical methods for systematically improving, designing, and understanding the electronic and optical properties of such structures. The interest in this field has been catapulted to a new height with the advent of epitaxial growth techniques, particularly molecular-beam epitaxy (MBE) and metalorganic chemical vapour deposition (MOCVD), through which fabrication of perfect superlattices and multi-quantum-well structures became a reality.

Since the pioneering work of Tsu and Esaki [1] resonant tunneling phenomena through multibarrier system (MBS) has generated considerable theoretical and experimental interest because of the potential and extensive applications of the resonant tunneling phenomenon in high-speed electronic and optoelectronic devices which include lasers, modulators, photodetectors, signal processing devices, etc. A notable feature of the tunneling through MBS is that it provides deeper understanding of the transport phenomena through semiconductor superlattices and similar structures like quantum-dot arrays. An understanding of the time dependent aspects of tunneling is clearly required for the construction of a kinetic theory for such systems.

The transmission coefficient (TC) is defined as the ratio of transmission intensity to the incident intensity. It can be considered as the relative probability of an incident electron crossing the system. Resonant tunneling in the MBS corresponds to unit transmission coefficient across the structure [7, 9]. One of the most striking features of the multi-barrier systems is the occurrence of quasi-level resonant tunneling energy states. Incident electrons with energies equal to any one of these quasilevel resonant energy states suffer resonant tunneling [6, 7]. Resonant tunneling is a consequence of the phase coherence of the electron waves in the quantum wells of the MBS. These quasi level resonant energy states group themselves into tunneling energy bands separated by forbidden gaps. Each allowed energy band comprises (N – 1) number of resonant energy states; N being the number of barriers in the MBS [7]. The resonant tunneling life time (RTL), is one of the important issues concerning the development of the novel electronic devices based on tunneling. During last decades the resonant tunneling lifetime (RTL) has stimulated the interest of several researchers [3 – 5, 9, 10, 13, 22]. A striking feature of RTL in the multi-barrier systems with more than two barriers is the occurrence of special minima in the resonant life time for quasi level resonant energies in the middle of the allowed tunneling energy bands of the MBS [10, 21]. Because the group velocity of an electron is the highest at the middle of an allowed band the resonant tunneling lifetime at the middle of allowed band assumes minimum value.

Most of the works on resonant tunneling reported so far are confined to single, double, triple or multibarrier cases without electric field and they use the methods based on envelope function approach to determine the resonant tunneling states. The few works where the MBS biased with electric field are studied have used the Airy function formalism with asymptotic forms to overcome the numerical flow of Airy functions at low bias condition [11]. We propose a study on tunneling in multibarrier systems biased with a varying electric field from low to high values by using an exact envelope function approach in the simple Airy function formalism that produce accurate result and help in the theoretical understanding of underlying physical principles [12, 14].

As mentioned earlier the RTL is especially useful for estimating the frequency limit and the operation speed of these high-speed devices. The investigations regarding tunneling through MBS have been mostly carried out in absence of electric field. The most striking feature of the tunneling through the MBS in absence of electric field is the occurrence of resonant tunneling energy states with transmission coefficient unity [7]. With the application of field, the current – voltage characteristics show negative differential conductivity regimes with resonant type anomalies similar to those seen in SLs for fields which favor Wannier Stark Ladder states [23]. Hence it is plausible to study the tunneling of electrons in electrically biased MBS and the electrical conduction in these systems. Some studies carried out in this direction show that resonant type energy states occur but the transmission coefficient takes values lower than unity [11, 12, 14]. The higher the applied electric field the lower the transmission coefficient the resonant states assume. However, the current – voltage characteristics exhibit resonance type structures with NDC regions in between the peaks that is in conformity with the results of Esaki and Tsu [1]. When the electric field beyond some threshold value applied to a particular MBS, the resonant states are destroyed [12]. In this situation the investigation on RTL through MBS in presence of electric field seems to be worthwhile and essential for estimating the properties of high speed solid state devices. So far no study has addressed this issue.

The electronic properties in condensed matter having heavy atoms require to be treated on relativistic footings. The electrons of such atoms may acquire velocities of the order 10^8 cm/s or more which are only two orders of magnitude smaller than the velocity of light in vacuum. So we may, as a matter of principle, expect relativistic impact on the electronic motion in case of such heavy atoms. Many researchers have been engaging in exploring the relativistic effect on the electronic properties. So far, the following aspects of condensed matter have received the relativistic treatment: (i) bulk states, (ii) surface states, (iii) interface states, and (iv) impurity states.

In the recent years the investigation of the lifetime of states corresponding to resonant tunneling has drawn considerable attentions [3 – 5, 9, 10, 13, 21]. While studying the resonant tunneling relativistically, it has been found that the resonant states occur in lower energies as compared to the non-relativistic (NR) ones [24 - 26]. But as per our knowledge, no investigation regarding the lifetime corresponding to the relativistic resonant tunneling sates has been carried out so far. So we have carried out a study on relativistic treatment in this direction. Briefly, our objectives are to (i) achieve a relativistic expression of the lifetime of the relativistic sates corresponding to resonant tunneling for an MBS, (ii) compare critically the relativistic results with NR results previously obtained [4, 13, 21], and (iii) elucidate the quantitative impacts of relativistic treatment on the lifetime of the resonant tunneling states.

Our relativistic treatment reveals all the basic features mentioned above remain unchanged, although a well controlled effect in magnitude of energy of resonance states and RTL has been observed [27]. But the presence of the electric field changes lot of aspects of resonant tunnelling, even the chance of occurring resonant tunnelling has been destroyed with application of certain amount of electric field.

In this perspective, we have carried out the investigation of the resonant tunneling lifetime through the semiconductor superlattices without and with electric field based on nonrelativistic as well as relativistic treatment in the present thesis.

A computational model based on theoretical investigations of field-free non-relativistic approach for resonant tunneling in the semiconductor superlattices is proposed for the determination of resonance energies across GaAs/$Al_yGa_{1-y}As$ for the energy range of $E < V_0$. V_0 being the potential barrier height. The effect of number of barriers and number of cells in the well and barrier regions on the resonant energies are studied in detail. Resonant tunneling is a consequence of the phase coherence of the electron waves in the quantum wells of this (MBS). The resonant energy states are found to group themselves into allowed tunneling bands separated by forbidden gaps. Each allowed energy band comprises (N – 1) number of resonant energy states; N being the number of barriers in the MBS. The resonant tunneling lifetime (RTL) has been determined by considering the half-width at half-maximum of the transmission peak for resonant tunneling. Specifically, our computational method employs a search technique to determine the values of the half-maximum of the resonant peak. We also find an analytical relation between the wave vectors and the resonant energy states in the tunneling energy bands. The RTL is found to have minimum values at the middle of the each allowed band. The resonant tunneling lifetime at the middle of allowed band assumes minimum value because the group velocity of an electron is the highest at the middle of an allowed band. Further, it is observed that the electrons with energies in the higher tunneling band could tunnel out faster than those with energies in the lower band.

We have investigated the relativistic effects on the tunneling of electrons through GaAs/$Al_yGa_{1-y}As$. A mathematical model based on relativistic Dirac equation has been employed for the determination of the relativistic resonance energy states within the energy range of $E<V_0$. The relativistic formula for transmission coefficient of a system of N number of rectangular barrier type potentials has been derived with the mass variation from the well to the barrier regions. This formula is compared critically with the corresponding non-relativistic one, especially in the context of resonant tunneling. The effect of number of barriers and number of cells in the well and barrier regions on the resonant energies are studied in details and we have discussed critically the quantitative extents of relativistic impacts on tunneling through such types of systems. A relativistic study on lifetime of resonant tunneling states for such systems has also been studied. This study reveals the fact that as compared to the non-relativistic resonant tunneling states the relativistic resonant tunneling states occur in the lower energies but they are having shorter lifetime.

The effect of uniform electric field on the resonant tunneling across the superlattice GaAs/$Al_yGa_{1-y}As$ is exhaustively explored by a computational method using exact Airy function formalism and the transfer matrix technique. The numerical computation takes care of the common problems of numerical inefficiency and overflow associated with the Airy functions for low applied voltages. Our investigation is basically concerned with the computation of (i) the transmission coefficient across MBS for incident energies, $E<V_0$, (ii) occurrence of resonant

energy states where the transmission coefficient is large but not equal to unity because of the presence of electric field, and (iii) resonant tunneling lifetime. In the unbiased periodic MBS the resonant tunneling corresponds to unit transmission coefficient across the structure. With the application of low and moderate dc-electric fields, the tunneling energy spectrum of the MBS gets Stark shifted and the transmittance for some of the resonance peaks gets lowered compared to those in the field free case due to gradual localization of wave functions which can be explained using bending band picture. The RTL is the most important parameters that have to be determined for realizing the tunable quantum cascade laser on the basis of Wannier-Stark ladder (WSL). The theoretical studies of RTL has been investigated for an electrically biased MBS, which would be the key issue in modeling high speed resonant tunneling devices. The resonance like states lifetime of those states vary significantly with applied electric field, mole fraction, well width and barrier width.

Moreover, since the experimental realization [28] of quasiperiodic semiconductor superlattices (SLs), such as Fibonacci [29] and Thue-Morse [30] SLs, there has been a growing interest both from experimental [31-34] and theoretical [35-37] viewpoints in the understanding of their unique physical properties, such as their quasiperiodic electronic and transport properties, as these systems represent intermediate cases between periodic and disordered solids. The electric-field-induced effects of an external homogeneous electric field *E* on the electronic structure [38] and properties [39] and optical properties [40] of quasiperiodic semiconductor SLs have also been extensively studied, both experimentally and theoretically.

An exhaustive study on the transmission coefficient and current density has been performed in case of an electrically biased GaAs-$Al_xGa_{1-x}As$ generalized Fibonacci superlattice. The study is based on transfer-matrix formalism using exact Airy function approach. This is the first report for the exact calculation of current density in case of quasiperiodic multibarrier systems. The present theoretical result suggests the use of such quasiperiodic systems in perfect band-pass or band-eliminator (of extremely low width) circuitry. It has also been found that there is a strong impact of quasiperiodicity on the current density as well as the negative differential conductivity for such systems.

The ordered systems are specified by their periodicity which permitted application of Bloch theorem. The case of disordered superlattices is more challenging because the translation by symmetry in this system is broken and the Bloch theorem is not applicable. We have studied theoretically the effect of short- range correlated disorder and applied bias on the nature of the transport properties in GaAs-$Al_xGa_{1-x}As$ superlattices. This is numerically examined with the use of the exact Airy function formalism and the transfer matrix method, the transmission properties across random Dimer, Trimer and Quaternary barrier superlattices. In the case of the unbiased systems, we observed that the introduction of correlated disorder prevent the localization and causes delocalization states. In the case of the biased systems, we see the decrease of the miniband width i.e reduction of transmission.

The fundamental physics underlying the relativistic resonance energies and resonant tunneling lifetime (RTL) in semiconductor superlattices with uniform electric field have been theoretically studied. The modeling of this relativistic resonance energies and RTL for biased superlattice

$GaAs/Al_yGa_{1-y}As$ has been developed based on Dirac equation on the framework of transfer matrix method and the half- width at half-maximum of the transmission peak. The resonance like states in the presence of electric field and their lifetime change as a function of electric field, mole fraction of the barrier layer, well width and barrier width. The relativistic resonance energies and the lifetime of the electrons at the resonant states under the action of uniform electric field are correlated with the band structure of the superlattices. It is seen that the variation of the relativistic RTL in presence of uniform electric field with resonance energy also has a special minima, and that these minima occur around the centre of the allowed bands. The field dependent relativistic study also reveals the fact that an electron with E_m (resonance energy) in the middle of the miniband would tunnel out faster than the ones with the values of E_m near the band edge.

We organize our thesis in the following way: Chapter – 1 is the Introductory Chapter where we describe our objectives and the way of presentation of our thesis. We review the earlier works on resonant tunneling and RTL in Chapter – 2. Chapter – 3 consists of a detailed study on RTL through semiconductor superlattices in both non-relativistic and relativistic framework. The changes in transmission coefficient , resonance energy states , and the lifetime corresponding to the resonance energy states with application of electric field to the semiconductor MBS have been discussed in Chapter – 4. It has been shown that the aspects like resonant energy, life time calculated in the relativistic framework almost tally with that obtained using non-relativistic framework. In chapter 5, all these aspects are studied for quasi periodic (generalized Fibonacci and generalized Thu-Morse) multibarrier systems. Chapter-6 includes the effects of the short range correlated disorder and applied bias on the nature of the transmission properties across random Dimer, Trimer and Quaternary MBS . Chapter–7 sums up the contents of whole thesis associated with our interesting observations and the conclusions related with the present investigation in a nutshell.

References

1. R. Tsu and L. Esaki, Appl. Phys. Lett, **22** (1973), 562.

2. B. Ricco and M. Y. Azbel, Phys. Rev B, **29** (1984), 1970.

3. P. J. Price, Phys. Rev B, **38** (1988), 1994.

4. C. J. Arsenault and M. Meunier, Phys. Rev B, **39** (1989), 8739.

5. J. W. Choe, H. J. Hwang, A. G. U. Perera, S. G. Matsik, and M. H. Francombe, J. Appl. Phys, **79** (1996), 7510.

6. C. L. Roy and A. Khan, Phys. Stat Solidi b, **176**(1993), K47 – K49.

7. C. L. Roy and A. Khan, Phys. Stat Solidi b, **176**(1993), 101.

8. C. L. Roy, Bulletin of Materials Science, **25** (2002), 469.

9. J. Nanda, P. K. Mahapatra, and C. L. Roy, Physica B, **383** (2006), 232.

10. A. Khan, P. K. Mahapatra, S. P. Bhattacharya, and S. N. Mohammad, Phil.Mag, **21** (2004), 547.

11. S. S. Allen and S. L. Richardson, Phys. Rev B, **50** (1994), 11693.

12. P. K. Mahapatra, P. Panchadhyayee, S. P. Bhattacharyaya, and A. Khan, Physica B, **403** (2008), 2780.

13. H. Xu, M. Zhu, B. Hou, Phys. Lett A **223**(1996), 227 - 231.

14. P. Panchadhyayee, R. Biswas, A. Khan, and P. K. Mahapatra, J. App Phys **104** (2008), 084517.

15. P. Pereyra, Phys. Rev. Lett, **84** (2000), 1772.

16. T. M. Kalotas and A. R. Lee, Phys. Stat Solidi b, **182**(1994), K5.

17. P. Erdos, E. Liviotti and R C Herndon, J. Phys. D, **30** (1997), 338.

18. S. Bentata, B. A. saadi, and H. Sedaki, Superlattices and Microstructures, **30** (2001), 297.

19. W. Salejda and M. H. Tyc, Physica A, **303** (2002), 493.

20. M. G. Tashkova and S. G. Donav, Phys. Stat Solidi b, **200**(1997), 75.

21. A. Khan and S. N. Mohammad, J. Appl. Phys, **92** (2002), 944.

22. A. Khan, P. K. Mahapatra and C. L. Roy, Phys. Lett A, **249** (1998), 512.

23. P. K. Mahapatra, K. Bhattacharya, A. Khan, and C. L. Roy, Phys. Rev B, **58** (1998), 1560.

24. C L Roy and A. Khan, Indian J. Pure and Appl. Phys, **30**(1992), 432.

25. C L Roy and A. Khan, J. Phys: Condens Matter **5**(1993), 770.

26. A. D. Alhaidari, H. Bahlouli, and A. Jellal, Advances in Mathematical Physics, vol. **2012**, Article ID 762908, 13 pages, 2012.

27. S. P. Bhattacharya, P. K. Mahapatra and A. Khan, Advances in Applied Physics, **1** (2012), 47.

28. K. W. Goossen, J. E. Cunningham and W. Y. Jan, Appl. Phys. Lett. **57** (1990) 2582.

29. R. Merlin, K. Bajema, R. Clarke, F.-Y. Juang, and P. Bhattacharya, Phys. Rev. Lett**. 55** (1985) 1768.

30. R. Merlin, K. Bajema, J. Nagle, and K. Ploog, J. Phys. (France) Colloq. **48** (1987) C5 503.

31. D. Munzar, L. Bocaek, J. Humlicek, and K. Ploog, J. Phys. Cond. Matter **6** (1994) 4107.

32. M. Dinu, D. D. Nolte and M. R. Melloch, Phys. Rev. B **56** (1997) 1987.

33. F. Dominguez-Adame, E. Macia, B. Mendez, C. L. Roy, and A. Khan, Semicond. Sci. Technol. **10** (1995) 797.

34. E. Diez, F. Dominguez-Adame, E. Macia and A. Sanchez, Phys. Rev. B **54** (1996) 16792.

35. P. E. de Brito, C. A. A. da Silva and H. N. Nazareno Phys. Rev. B **51** (1995) 6096.

36. F. Laruelle, B. Etienne, J. Barrau, K. Khirouni, J. C. Brabant, T. Amand and M. Brousseau, Surface Science **228** (1990) 92.

37. M. Castro and F. Dominguez-Adame, Physics Letters A **225** (1997) 321.

38. E. Reyes-Gómez, C A Perdomo-Leiva, L E Oliveira and M de Dios-Leyva, J. Phys. Cond. Matter **10** (1998) 3557.

39. C. L. Roy and A. Khan, Phys. Rev. B **49** (1994) 14979.

40. E. Macia, F. Dominguez-Adame, and A. Sanchez, Phys. Rev. B **49** (1994) 9503.